Social Beliefs and Stock Price Booms and Busts

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Abstract

I develop a dynamic asset pricing model in which subject beliefs about stock price behavior are heterogeneous and susceptible to peer effects. Two types of traders optimally learn from past price realizations and share beliefs in a social network. I show that, at each period, the equilibrium price is a function of traders' beliefs and the network structure. As a result, booms and busts of the price-dividend ratio emerge. The most (least) speculative trader is the most influential during booms (busts). More connected networks exhibit less volatile price dividend ratio, booms and busts episodes last longer, and the average price realization is higher. Also, there is less disagreement in beliefs. However, if traders of the same type are highly interconnected stock market volatility is higher and booms and busts are shorter. The model captures relevant empirical features of stock prices and returns, and it is also consistent with the survey evidence on investor expectations.

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1 Introduction

Motivated by recent booms and busts episodes in several asset markets, research has been devoted to better understand why and how such behavior emerges. At the same time, it is known that information and beliefs play a role in every model and every economic choice. The existing literature on asset pricing mainly focus on agents having subjective beliefs about market outcomes as an explanation for such phenomena of aggregate stock market prices. Yet, the social structures where these agents directly interact with one another have not been extensively explored. Social connections transmit information through communication, exchange of opinions, and observations of others' decisions. In this paper I argue that, along with heterogeneity in beliefs, information sharing dictated by investors' social ties are an important ingredient for understanding asset price dynamics.

I explore the effects of communication in an asset pricing model introduce by Adam et al. (2016) in which agents learn about stock-market price and are internally rational. I extend their work by allowing agents to hold heterogenous subjective expectations about asset price behavior and diffuse beliefs through connections captured by an exogenous social network of investors. At each period, agents hold private beliefs which are susceptible to social influece. After interacting with others in their neighborhood, they formulate public beliefs, which are a result of peer effects excerced by whom the agents are connected to. Hence, social interaction renders a conformism in public beliefs.

I focus on the role of social network as facilitating social influence, specifically through communication. People are constantly talking with each other, discussing opinions and seeking knwoledge. For instance, Calvó-Armengol et al. (2009) and Jackson (2010) argue that the behavior of individual agents is affected by that of their peers. This 'peer effect' can shape human behavior in the sense that a person holds certain public beliefs because his friends do so. Specifically in the financial markets, one can observe that investors tend to seek financial advice, search for stock-market news and discuss with their mates before making asset investment decisions. It is then natural to think that investors incorporate, at least partially, some of this social engagement into their conduct towards economic activity and tend to conform with those who they interact. Several studies in the network literature have explored these concepts under a wide range of research area (To name a few: Bala and Goyal (1998), DeMarzo et al. (2003), Banerjee et al. (2013), Banerjee and Fudenberg (2004), Golub and Jackson (2010)).

In the model, agents formulate beliefs about risk-adjusted stock price growth using optimal Bayesian filtering techniques and last-period observations of price and dividend. They adjust beliefs upward (downward) if they underpredict (overpredict) the realized price outcome. I characterize agents into two types, experienced (L) and inexperienced (H) investors. Both types have the same optimal belief-updating equation except that H-agents update their posteriors more heavily in response to forecast errors. Hence, inexperienced agents have more volatile expectations than experienced ones. Evidence from U.S. stock-market (Vissing-Jorgensen (2003), Adam et al. (2015)) shows that investors with less experience in the stock market (in number of years) updated expectations about future asset returns more strongly. This fact can be explained, for example, by investors having different information sources. In this case, experienced agents, who have been trading for longer, have access to certain pieces of valuable information before other investors; while the inexperienced type rely more on publicly available information such as past asset prices. Another plausible reason is that those with more experience have higher self-confidence to trade and so rely less on last period observations. Finally, investors can simply have different economic models in mind that lead them to interpret forecast errors diferently. Hong and Stein (2007) extensively discuss sources of disagreement in expectations in the stock market and corroborate their arguments with evidences from U.S. data.

After the individual learning scheme takes place, investors engage in direct, truthfull communication with others that are connected to them through an underlying social network. Each agent attaches a weight to each belief he receives, and these weights are type-specific. Hence, invidual weights are given by the proportion of H-type and L-type friends each investor has. This information sharing results in a new revision of subjective expectations. However, communication does not change agents' private belief about price growth, given by his type, and so agents agree to disagree forever.

Private and public beliefs interplay between each other. Each agent's private belief is given by his type and determines the way the agent filters out information from past observed market outcomes. This is an individual trait of the agent. In addition, each investor is influenced by his neighbors and he has incentives to conform to his friends' patterns of behavior. In the presence of such peer pressure, an agent formulates his public opinion by averaging his friends' beliefs. There is a large body of literature on the subject of peer effects in different areas, such as economics, sociology, education and crime. One challenge is how to capture such social influence. I choose a simple average of friends' beliefs for simplicity but the model accomodates other sorts of weighted averages. For instance, one alternative is De Groot Learning (DeGroot (1974)), in which each agent has a social influence weight.

The timing of the model is as follows. At the beggining of each period and using last-period observations, agents update private (initial) beliefs based on their forecast error. Then, agents share private beliefs with others who are part of their neighborhood and update their beliefs according to the weight their linkages imply. This results in investors' public beliefs. At the end of the period, equilibrium is determined by agents exchanging stock holdings and, since the stock market is perfectly competitive, the agent with the highest public belief holds the asset and determines it price.

I show that, because agents formulate and share beliefs about asset price, the equilibrium is then determined by investors' expectations. Differently from Adam et al. (2016), equilibrium price is a function of only the most optimistic agent's belief, defined as the one with the highest public belief.The only sources of heterogeneity among agents concern their perception about future stock-market behavior and who they are connected to. Since the stock market is perfectly competitive, in equilibrium the asset is held by the investor who is willing to pay the most for it which is precisely the agent with the highest expectation about asset price growth.

The model gives rise to a feed-back loop between beliefs and realized price, resulting in booms-and-busts phenomena of the price-dividend ratio. When asset price is increasing, both types expect it to keep increasing, what feeds back into a higher equilibrium price. This further increase in price makes agents revise beliefs upwards again, and realized price is again higher, and so on. The reverse holds during a bust. This self-referential aspect is similar to Adam et al. (2016). However, in my model the extent and frequency of booms and busts depend on the social network. I also obtain different behavior of agents' public beliefs which is a function of their network structure.

I show that booms episodes are determined by agents with more social connections to inexperienced investors. When price is increasing, and given that inexperienced agents are more reactive, H-type initial beliefs are higher than L-type beliefs. Thus, agents who interact more with inexperienced investors become more optimistic and so have higher public beliefs. The contrary holds during busts, marked by experienced agents being more optimistic. The periods inbetween boom and busts, referred as 'recovery periods', are characterized by the stock price staying around its mean value and experienced agents are the most influential, since H-type beliefs become too pessimistic after a bust.

The importance of modeling information sharing as weighting neighbors' beliefs is two fold. First, it captures the network structure. Second, equilibrium is a function of private beliefs. Even though only the most optimistic agent determines asset price, his public belief is a combination of others' beliefs who are socially linked to him. Hence, the fluctuations of the PD ratio hinge on the existence of correlated movements in agents beliefs and the social effects agents are exposed to.

I find that the structure of the social network matters for asset price fluctuations and price-dividend volatility. More connected networks exhibit less volatile price-dividend ratio and booms and busts episodes last for longer periods. At the same time, in such structures there is less dispersion in public beliefs and the average price realization is higher. Nonetheless, if agents of the same type are more connected to each other, stock market price fluctuates more and booms and busts have shorter duration.

I also evaluate the quantitative performance of the model by simulating the economy with different network structures. As Adam et al. (2016), I find that the model can replicate key empirical facts, specially the high volatility of the PD ratio and excessive return volatility. The equity premium puzzle, which they have difficulty in matching, I cannot also fully replicate. Overall, my model can account approximately half of the observed equity premium, what is slightly above Adam et al. (2016)' finding. Due to the lack of data, I am not able to confront simulation results with other countries' stock market, such as Japan and te Euro-Area. Nonetheless, Adam et al. (2010) report evidence that these countries, even though they all have experienced boom and busts, the frequency and timing of these cycles, as well as the variation of the PD ratio differ across them. I argue that one of the many reasons of why this is the case is their different social structures. I see this as a next research avenue.

As a final exploration, I relax the assumption of the social network being exogenously determined. I use a random graph approach, widely used in the network literature (Jackson and Wolinsky (1996), ERDdS and R&wi (1959)), to infer what network would emerge if the only knowledge one has is the probabilities of agents to connect with each other. I investigate the impact of homophily (Golub and Jackson (2012)) in stock market-equilibrium, that is, I assume agents of the same type tend to share more connections. I assume the network structure is captured by the homopholy index and that equilibrium is then a function of homophily and private beliefs. I find that asset price volatility and the frequency of booms and busts are positively related to homophily, whereas the duration of such episodes is negatively related.

I provide a simple way to introduce social interaction into a standard asset pricing model. The results show that stock market outcomes - namely, equibilibrium asset price, price-dividend behavior and agents' subjective beliefs - are all dependent on the underlying network topology. To my knowledge, this paper is the first to combine adaptive learning and social networks in the asset pricing literature. For instance, research on asset pricing and learning, in which Adam et al. (2016) fits in, has not yet incorporate any form of social dynamics or information sharing. Meanwhile, stock market models in information networks such as Ozsoylev and Walden (2011) and Xia (2007) all share the standard rational expectations assumption. Thus, this paper can serve as an initial step to better comprehend the interdependency of the stock-market behavior and the social structure of the economy.

Related Literature: My work relates to two fields of ongoing research about stock market behavior: adaptive learning and asset pricing in networks.

Under the adaptive learning literature, agents learn about stock prices. The underlying main hypothesis is that agents are not 'perfectly rational' in the sense of rational expectations (RE) but instead do not know exactly the pricing function that governs asset price and so they take rational decisions given the information they are assumed to possess. Adam and Marcet (2011) are the first to provide a microfoundation for these models and introduce the concept of 'internally rational' agents who maximize discounted expected utility under uncertainty given dynamically consistent subjective beliefs about the future. They study a simple asset pricing model with risk-neutral investors and show that learning about price behavior affects market outcomes, while learning about the discounted sum of dividends, the case under RE, is irrelevant for equilibrium prices.

In this same line, Adam et al. (2016) study a simple variation of the Lucas Jr (1978) asset pricing model in which agents hold subjective prior beliefs about risk-adjusted price growth. They show that internally rational agents update their beliefs about stock price behavior using observed stock price realizations. The setup gives rise to a model characterized by large asset price fluctuations and volatile price dividend ratio. They are able to replicate some stylized facts of U.S. stock price data that previous RE models have not been able to account for. Namely, the high persistence and volatility of the price-dividend (PD) ratio; high volatility of stock returns; and the predictability of long-horizon excess stock returns. As they explain, these results arise because there is a momentum of changes in beliefs around the RE value and a mean-reverting behavior of beliefs. Nonetheless, their model fail to reproduce the observed equity premium under reasonable degrees of risk aversion.

My model introduces an additional dimension to Adam et al. (2016). I let investors be heterogenous in their beliefs about asset price behavior and assume the existence of a social network connecting all agents. Given their linkages, agents are able to communicate and share individual beliefs about risk-adjsuted price growth. In this sense, the present framework captures how different stock price expectations and social interaction can affect equilibrium asset pricing outcomes and agents market behavior.

The asset pricing literature has documented the relevance of heterogeneity and social interaction among investors as influential to observed stock market outcomes. Vissing-Jorgensen (2003) surveys evidence on the stock market behavior and actions. She documents that an investor's belief about future stock-market returns depends on the investor's own experience measured by age, years of investment experience, and own past portfolio returns. She shows that expected returns are higher for those with low investment experience for a given age than for those with more years of experience. The evidence also suggests that investor beliefs do affect their stockholdings, and she concludes that that understanding beliefs is in fact useful for understanding prices.

Adam et al. (2010) report evidence that different stock-markets historically experienced substantial and sustained price increases that were followed by sustained and long lasting price reversals. They show time-series data for the U.S., Japan and the Euro-Area since 1970 and all markets present PD ratio booms and busts. However, the frequency and timing of these cycles, as well as the variation of the PD ratio differ across them. I argue that one of the many reasons of why such pattern of the PD ratio is different among those countries can be their social structure.

My work also fits in the strand of research about asset pricing in networks. Ozsoylev and Walden (2011) study the impact of the properties of information networks on asset pricing in a rational expectations equilibrium model. Agents communicate information to each other about asset payoffs according to an exogenous information networt and each agent has some information about her network neighbors payoff-related information. They provide closed form solutions and find that aggregate properties of the market - for example, price volatility, expected trading profits and agent welfare - are non-monotonic functions of network connectivity. Hong et al. (2005) use U.S. data on mutual fund holdings from the late 1990's and find that a mutual fund manager is more likely to buy (or sell) a particular stock in any quarter if other managers in the same city are buying (or selling) that same stock. They argue that this can be seen as the result of an epidemic model in which investors spread information about stocks to one another by word of mouth. However, their work is purely empirical and they do not estimate the underlying network per se. Xia (2007) develops an strategic rational expectations asset pricing model with one-way direct and truthful information transmission in circle and star networks. He finds that several aggregate market outcomes, including trading volume and price volatility, are all higher with communication than without.

The aforementioned studies share the same underlying assumption of rational expectations, which essentialy means that all agents have perfect knowledge about the pricing function that maps fundamentals to asset price. On the contrary, the agents in my model are 'internally rational' and hold a perceived law of motion for asset price while the realized price derives from an actual ('true') law of motion.

The role of communication, and more broadly social interactions, through social network in economic models has been recognized in other fields. Bala and Goyal (1998) show that communication in social networks play a major role in technology adoption. Banerjee et al. (2013) show, theoretically and empirically, that information diffusion influences people's adoption decision, in their case, in the participation of a microfinance program. Golub and Jackson (2012) explore how the speed of convergence of agents behaviors and beliefs depends on network structure. DeMarzo et al. (2003) consider the spread of information across a given network when individuals are subject to persuasion bias and show that agents with particular network positions can have disproportionate influence. Jackson (2010) presents a textbook on network economics.

I contribute to the stock-market behavior literature by aligning adaptive learning and social interaction. I introduce a simple model of communication among socially connected investors in an otherwise stadard asset pricing model where agents learn about prices. I derive the equilibrium outcomes of the model as a function of the communication strucutre. Also I quantitatively evaluate its perfomance to show that the model can replicate the observed asset market booms and busts and that the characteristics of such cycles vary considerably depending on social network.

The paper is organized as follows. Section 2 presents the model and Section 3 defines the equilibrium. Section 4 discusses agents' learning problem. Section 5 outlines the solution of the model and characterizes agents' behavior. Section 6 explores the implications for asset price dynamics. Section 7 discusses equilibrium as a function of network characteristics. Section 8 presents the results of simulating the model. Section 9 extends the model to the case with an unkown network structure. Section 10 concludes. The

Appendix contains omitted proofs and discussions.

2 A Network Asset Pricing Model

I introduce an extended version of the model studied by Adam et al. (2016). It is a Lucas Jr (1978) asset princing model in which agents hold heterogenous prior beliefs about stock price behavior and are a part of an exogenously given social network. Those who are socially connect share beliefs. The information sharing is truthfull and credible (agents commit not to lie), and agents do not act stratigically towards price manipulation in ones' favor.[1] Agents are susceptible to peer effects from those thet are connected to.

2.1 Model Enviroment

Consider a discrete-time economy with a large number of N of infinitely-lived agents trading one unit of stock in a competitive stock market. At each period a stock yields a stochastic divend D_t and investors receive an exogenous endowment Y_t, both in the form of perishable consumption goods.

Given Y_t and D_t, aggregate consumption is such that economy's feasibility constraint holds: $C_t = Y_t + D_t$.

The exogenous processes for dividends and aggregate consumption are, respectively

$$\frac{D_t}{D_{t-1}} = a\epsilon_t^d, \quad \log \epsilon_t^d \sim_{ii} N(-\frac{s_d^2}{2}, s_d^2) \tag{1}$$

$$\frac{C_t}{C_{t-1}} = a\epsilon_t^c, \quad \log \epsilon_t^c \sim_{ii} N(-\frac{s_c^2}{2}, s_c^2) \tag{2}$$

where $a \geq 1$ and $\log \epsilon_t^d, \log \epsilon_t^c$ are jointly normal. Let $\rho = E\Big[(\epsilon_{t+1}^c)^{-\gamma}\epsilon_{t+1}^d\Big] = \exp\Big(\gamma(1+\gamma)\frac{s_c^2}{2}\Big)\exp\Big(-\gamma\rho_{c,d}s_c, s_d\Big)$.

There are two types of investors $K = \{h, l\}$: inexperienced (H) and experienced (L). Types differ in their private expectations about the behavior of future stock prices: $H-$agents have more volatile private beliefs than $L-$agents.[2] This characterization is motivated by

[1] That is, agents do not wish to formulate beliefs in order to influence price in such a way to make them hold asset in equilibrium.

[2] I further disucss the characterization of types in Section 4

Adam et al. (2015), who documented evidence that investors with less stock market experience are more heavily influenced by recent asset price realizations than those with more years as traders.

The investment problem

The risk-averse and internally rational agents have standard time-separable consumption preferences. Investor $i \in N$ of type $k \in \{h, l\}$ chooses consumption C_t^{ik}, bonds B_t^{ik} and stock holdings S_t^{ik} in order to maximize expected future utility[3]:

$$\max_{\{C_t^{ik}, S_t^{ik}, B_t^{ik}\}_{t=0}^{\infty}} E_0^{ik} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^{ik})^{1-\gamma}}{1-\gamma} \tag{3}$$

$$S_t^{ik} P_t + C_t^{ik} + B_t^{ik} \leq S_{t-1}^{ik}(P_t + D_t) + (1 + r_{t-1}) B_{t-1}^{ik} + Y_t \tag{4}$$

where $\gamma \in (0, \infty)$ is the coefficient of relative risk aversion; r_{t-1} is the real interest rate on riskless bonds issued in period $t-1$ and maturing in period t; and E_0^i denotes agent's subjective probability space that assigns probabilities to all external variables P_t, D_t, Y_t.

Initial endowments of individual stock holdings and bonds are $S_{-1}^{ik} = 1, B_{-1}^{ik} = 0$. To avoid Ponzi schemes and to guarantee a solution to the problem, assume the following bounds hold, $\forall i \in N, k \in K$:

$$\underline{S} \leq S_t^{ik} \leq \bar{S} \tag{5}$$

$$\underline{B} \leq B_t^{ik} \leq \bar{B} \tag{6}$$

with $\bar{S}, \bar{B} < \infty$, $\underline{S} < 1 < \bar{S}$ and $\underline{B} < 0 < \bar{B}$.

Below I will specify agents' subjective probabilities by a learining scheme describing their view about the evolution of (Y_t, D_t, P_t). Differently from what is implied by the RE assumption, these subjective probabilities may or may not coincide with the true probabilities governing the behavior of those variables.

Agent's optimal choices are given by the first order conditions of his utility maximization problem:

[3]For simplification, I assume agents have the same discount factor δ and coefficient of relative risk aversion γ.

$$(C_t^{ik})^{-\gamma} P_t = \delta E_t^{ik}\Big[(C_{t+1}^{ik})^{-\gamma} P_{t+1}\Big] + \delta E_t^{ik}\Big[(C_{t+1}^{ik})^{\gamma} D_{t+1}\Big] \tag{7}$$

$$(C_{t+1}^{ik})^{-\gamma} = \delta(1+r_t) E_t^{ik}\Big[(C_{t+1}^{ik})^{-\gamma}\Big] \tag{8}$$

As in Adam et al. (2016), I assume agent's individual income Y_t is high enough and that expected future PD ration is bounded.[4]

Given the individual maximization problem above, assume agents' private wealth Y_t is sufficiently large and that $E_t^{ik}\frac{P_{t+1}}{D_t} < \bar{M}$ for all i, k and for some $\bar{M} < \infty$, such that individual consumption choices can be approximated by the aggregate consumption: $\frac{C_{t+1}^{ik}}{C_t^{ik}} \approx \frac{C_{t+1}}{C_t}$

Hence, the first-order conditions boil down to

$$(C_t)^{-\gamma} P_t = \delta E_t^{ik}\Big[(C_{t+1})^{-\gamma} P_{t+1}\Big] + \delta E_t^{i}\Big[(C_{t+1})^{\gamma} D_{t+1}\Big] \tag{9}$$

$$(C_{t+1})^{-\gamma} = \delta(1+r_t) E_t^{ik}\Big[(C_{t+1})^{-\gamma}\Big] \tag{10}$$

Define subjective expectations of risk-adjusted stock price growth and dividend growth to be, respectively

$$\beta_t^{ik} \equiv E_t^{ik}\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{P_{t+1}}{P_t}\right] \tag{11}$$

$$\beta_t^{d,ik} \equiv E_t^{ik}\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{D_{t+1}}{D_t}\right] \tag{12}$$

Stock holdings first-order condition (9) can then be written as

$$P_t = \delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma} E_t^{ik}\Big[P_{t+1} + D_{t+1}\Big] = \delta\beta_t^{i} P_t + \delta\beta_t^{d,ik} D_t \tag{13}$$

RE as a special case

When one assumes agents know the exact mapping from both exogenous processes (D_t, Y_t) into equilibrium asset pricing $P_t(Y^t, D^t)$, beliefs and equilibrium are given by

[4]This is the *Assumption 1* on Adam et al. (2016). Please refer to it for further details.

$$\beta_t^{ik} = \beta_t^{d,ik} = \beta^{RE} \equiv a^{1-\gamma}\rho \tag{14}$$

$$P_t = \frac{\delta\beta^{RE}}{1-\delta\beta^{RE}} D_t \tag{15}$$

Hence, under RE, beliefs about both risk-adjusted price growth and divdend growth are constant and so it is the price-dividend (PD) ratio. Asset price follows the same stochastic process as dividend and the PD ratio is constant.

2.2 Types' Private Beliefs

In order to focus on subjective expectations of stock market price, I endow agents with perfect knowledge of risk-adjusted dividend growth, that is, all agents hold rational expectations about these dividend processes:

$$\beta_t^{d,ik} = \beta^{RE} \quad \forall i \in N, k \in K, \forall t \tag{16}$$

Agents also know the consumption stochastic process. However, they do not know how the equilibrium asset pricing function and they must formulate beliefs about risk-adjusted price growth at every period. This implies that each agent has a subjective (and possibly different) view about the price dividend ratio at each period.

Heterogeneity is characterized by different types having different private beliefs. That is, $\beta_t^{ik} = \beta_t^{jk}$ for $i \neq j, \forall t$ and $\beta_t^{ik} \neq \beta_t^{jk'}$ for $i \neq j, k \neq k', \forall t$. Denote beliefs of inexperience and experience agents, respectively, as β_t^h and β_t^l.

2.3 The Social Network

Investors are located in an exogenously given network G of N nodes, each representing an agent. The relation $\mathcal{E} \subset N \times N$ describes which investors are connected in the network. $(i,j) \in \mathcal{E}$ means there is an edge (link) between agents i and j. I assume connections are bidirectional, $\mathcal{E}_{ij} = \mathcal{E}_{ji}$, so that $\mathcal{E}$ is symmetric.

The distance between two agents i and j is represented by a function $d^{\mathcal{E}}(i,j)$, which defines the number of edges in the shortest path between agents i and j. I use the convention that $d^{\mathcal{E}}(i,j) = \infty$ if there is no path between the two agents. An agent neighborhood N_{ik} is set of his closest nodes: $N_{ik} = \{j : d^{\mathcal{E}}(i,j) = 1\}$. I shall refer to a node in an agent's neighborhood as a friend. The degree of investor ik is defined as the investor's number of neighbors, including himself: $D_{ik} = |N_{ik}|$.

2.4 Social Dynamics

The main concern of this paper is to explore the effects of the network structure in stock market price behavior. Investors are susceptible to social influence. They exchange information about asset price through direct communication with those they are linked to. Agents conform with their peers and so revise private beliefs after social interaction.

At the beggining of each period, each agent ik updates his type-specific private belief β_t^k, $k = \{h, l\}$. Then, he interacts with others who belong to his neighborhood N_{ik}. Let the weight an investor ik assigns to type-H private beliefs β_t^h be λ^i and the one of type-L beliefs β_t^l be $(1 - \lambda^i)$. Intuitivelly, λ^i captures the social influence of different types in investor ik's behavior. λ^i is a function of the investor's social network, not of his type k, and it is determined by the proportion of inexperienced friends he has:

$$\lambda^i = \sum_{j \in N_{ik}} \frac{1\{j = H\}}{D_i} = \frac{D_i^h}{D_i} \tag{17}$$

where $D_i^h = \{\#\text{H friends}\}$. Notice that λ^i accounts for investor's own type (self-loop).[5]

After communication, agent formulates a public belief by averaging their neighbors' expectations. Hence, at the end of each period agent's risk-adjusted price growth belief is given by

$$\tilde{\beta}_t^i = \lambda^i \beta_t^h + (1 - \lambda^i)\beta_t^l \quad \forall i \in N, k \in K \tag{18}$$

$\tilde{\beta}_t^i$ is agent ik's public belief. Differently from his private belief, $\tilde{\beta}_t^i$ depends on the investor's set of friends, and not so much on his own-type. To make this point clearer, the superscript i from β_t^k , and the supersript k from $\tilde{\beta}_t^i$ are dropped.

Social dynamics is then incur peer effects and agents hold certain public beliefs because their friends do so. The challenge of the model is then how social interaction and influence affect stock price behavior.

[5]The λ^i measure includes agent's own type to capture the idea that individual beliefs matter. So that agents' subjective expectations are not solely determined by who they are connected to but also depends on they own type.

3 Equilibrium

Since agents have heterogenous beliefs and do not hold rational price expectations, the stochastic process for equilibrium price is different from agents' perceived price process. The latter is given by individual stock-holdings first-order condition (13). Since the asset market is perfectly competitive, agents exchange stock holdings and the the asset is held by the agent who is willing to pay the most for it. Hence, equilbrium is determined by the marginal agent who holds the most optimistic public belief $\tilde{\beta}_t^i$..[6]

Denote the most optimitic's subjective belief about risk-adjusted stock price growth as

$$\tilde{\beta}_t^o = \max_{i \in N} \tilde{\beta}_t^i \tag{19}$$

The equilibrium pricing function is then[7]

$$P_t = \frac{\delta a^{1-\gamma}\rho}{1-\delta\tilde{\beta}_t^o} D_t \tag{20}$$

where $a^{1-\gamma}\rho \equiv \beta^{RE} = \beta_t^d$, $\forall t$. This equation implies that agents' beliefs explicity determine equilibrium price. Asset price is increasing in both subjective expectations about risk-adjusted price growth and dividend growth.

For realized price to be well-defined at all periods, for any set of beliefs, I impose the existence of a maximum price-dividend value. This is justified by the fact that PD ratio will be bounded in equilibrium and it is consistent with the behavior of internally rational agents, and with asset pricing data..[8]

There exists a maximum equilibrium price-dividend ratio $\bar{PD} < \infty$.

4 The Learning Problem

In order to formulate subjective beliefs about the stock price behavior, investors make use of two learning channels. Firstly, they use last-period price and dividends observa-

[6] Adam and Marcet (2011) extensively discuss this result under a similar framework. Please refer to them for more details.

[7] To see this, I can write

$$P_t = \max_{i \in N} \left[\delta E_t^i \left(P_{t+1} + D_{t+1} \right) \right]$$

and substitute out for $\tilde{\beta}_t^o$.

[8] See the disucssion under Adam and Marcet (2011) and Adam et al. (2017).

tions applied to Bayesian filtering techiniques to infer about realized stock market outcomes. Secondly, they share beliefs with their friends.

The former channel pins down private beliefs β_t^k, while the later determines public beliefs $\tilde{\beta}_t^i$. Social interaction has been introduced in the previous section, and to close up the model the individual learning scheme is specified below.

Recall that private beliefs are a function of agents' type and so there are as many β_t^k as existing types in the economy. Each type, and therefore agent, perceives that the process for risk-adjusted stock price growth is the sum of a persistent component b_t^k and a transitory component ε_t^k ,

$$\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma} \frac{P_t}{P_{t-1}} = b_t^k + \varepsilon_t^k \tag{21}$$

$$b_t^k = b_{t-1}^k + \xi_t^k \tag{22}$$

for $k = h, l$ where $\varepsilon_t^k \sim_{ii} N(0, \sigma_{\varepsilon,k}^2)$ and $\xi_t^k \sim_{ii} N(0, \sigma_{\xi,k}^2)$, such that $\sigma_{\varepsilon,h}^2 > \sigma_{\varepsilon,l}^2$ and $\sigma_{\xi,h}^2 > \sigma_{\xi,l}^2$.

Let types' prior beliefs b_0^k be centered around the RE belief[9]

$$\begin{aligned} b_0^k &\sim_{ii} N(a^{1-\gamma}\rho, \sigma_{0,k}^2) \\ \beta_0^h &= \beta_0^l \\ \sigma_{0,k}^2 &= \frac{-\sigma_{\xi,k}^2 + \sqrt{(\sigma_{\xi,k}^2)^2 + 4\sigma_{\xi,k}^2\sigma_{\varepsilon,k}^2}}{2} \end{aligned} \tag{23}$$

where $\sigma_{0,k}^2$ is the steady-state Kalman filter uncertainty about b_t^k for $k = h, l$. Type's posterior belief at any time period t is distributed as

$$b_t^k \sim N\left(\beta_t^k, \sigma_{0,k}\right) \quad \forall k = h, l \tag{24}$$

where $\sigma_{0,h} > \sigma_{0,l}$.

To learn about asset price growth, agents act as Bayesians and optimality filter out the persistent component b_t^k. Posterior beliefs are then given by

[9]Following Adam et al. (2016), our framework constitute a small deviation from the RE case.

$$\beta_t^k = \beta_{t-1}^k + g^i \left[\left(\frac{C_{t-1}}{C_{t-2}} \right)^{-\gamma} \frac{P_{t-1}}{P_{t-2}} - \beta_{t-1}^k \right] \tag{25}$$

$$g^k = \frac{\sigma_{0,k}^2 + \sigma_{\xi,k}^2}{\sigma_{0,k}^2 + \sigma_{\xi,k}^2 + \sigma_{\varepsilon,k}^2} \tag{26}$$

where g^k is the optimal Kalman gain and reflects how the type reacts to his forecast error $e_{t-1}^k \equiv \left(\frac{C_{t-1}}{C_{t-2}}\right)^{-\gamma} \frac{P_{t-1}}{P_{t-2}} - \beta_{t-1}^k$. Comparing types learning scheme, H-agents update more heavily their beliefs at each period and have more volatile expectations .

This setup means that both β_t^h, β_t^l constitute a small deviation from RE beliefs in the limiting case with vanishing uncertainty about the persistent component b_t ($\sigma_{0,k}^2 \to 0, \forall k$). In such scenario types share the same belief which converges in distribution to β^{RE}. However notice that RE beliefs are never actually attainable and agents are consistently wrong on predicting price growth.

5 Solving the Model

It is important to keep in mind some important features of the model i) agents are internally rational and, at every period, make inferences about the stock market behavior by formulating subjective risk-adjusted stock price growth beliefs; ii) at each period, agents' learning problem consists of two stages: an individual Kalman filter problem that pins down private beliefs (type-specific) β_t^k; and communication, which pins down public specific (agent-specific) $\tilde{\beta}_t^i$; iii) equilibrium price is determined by the most optimistic agent, defined as the agent with the highest public belief $\tilde{\beta}_t^o$.

Henceforth, the model timeline is:

1. at t, last-period beliefs $\beta_{t-1}^i, \tilde{\beta}_{t-1}^k$ for $i \in N$, $k = h, l$ are determined and agents observe realized price P_{t-1} and dividends D_{t-1};

2. still at t and using past price observations, each type solves its learning problem by applying Kalman filtering techniques. This pins downs time t private beliefs β_t^h, β_t^l.

3. then and still at t, agents communicate and public beliefs are set: $\tilde{\beta}_t^i, \forall i \in N$;

4. at the end of period t, equilibrium price P_t is determined by the most optimistic agent: $\tilde{\beta}_t^o$;

5. at $t+1$, this process is repeated.

Notice that agents beliefs $\beta_t^k, \tilde{\beta}_t^i$ are predetermined at time t, so that the economy evolves according to a uniquely determined recursive process: the market-clearing price for period t is determined given time t beliefs; then in the next period $t+1$, beliefs are updated following this observation, communication takes place and $t+1$ PD ratio is realized.

Even though the most optimistic agent sets price, both types' private beliefs $\beta_t^k, k = h, l$ directly impact equilibrium PD ratio. This is a novelty introduced by this framework compared to the baseline model studied by Adam et al. (2016). Communication plays an important role since it determines the extent of each type's influence on realized risk-adjusted stock price growth: the type-belief with a higher weight (λ^i or $1-\lambda^i$) will have a higher impact on the price-setting equilibrium belief.

Solving the model consists on basically two stages, which are mutually influential: 1) the Kalman filter problems, which deliver β_t^h, β_t^l - this is essentially what Adam et al. (2016) do and I refer to them for the details of the results; 2) determine equilibrium weight λ_t^*, which delivers equilibrium beliefs and price $\tilde{\beta}_t^o(\lambda_t^*)$, $P_t(\lambda_t^*)$ - this is done by analyzing the network structure.

5.1 First Stage: Individual Learning Problem

The individual learning problem entails different evolution of belief depending on agent's type. In order to study the dynamics of the model it is then crucial to know how beliefs change through time and how distinct are these processes.

Bounded Beliefs

First-order condition (13) implies that subjective private beliefs must be bounded: to guarantee a solution for individual opimization problem it must hold that[10] $\beta_t^k < \delta^{-1} \quad \forall k, t$. Intuitively, private beliefs must not be too optimistic. Overly optimistic beliefs can give rise to a situation where subjective expected utility is infinite, so that problem agents' first order condition does not have a well defined solution. In turn, bounded beliefs guarantee the existence of a finite equilibrium PD ratio. Denote beliefs' upper bound by β_k^U.

[10]To see this notice that if $\beta_t^k = \delta^{-1}$, then $\delta \beta^{RE} D_t = 0$ which means there exists infinetely-many solutions to the first-order contion. On the other hand, if $\beta_t^k > \delta^{-1}$, then $\delta \beta^{RE} D_t < 0 \to D_t < 0$ which cannot hold.

For all $i \in N$, $k = \{h, l\}$ and t with $P_t < \infty, D_t < \infty, C_t < \infty$, subjective expected risk-adjused price growth is such that

$$\beta_t^{ik} < \beta^U \tag{27}$$

where $\beta_l^U = \beta_h^U \equiv \beta^U < \delta^{-1}$.

Comovement of Beliefs

Intial beliefs β_t^k are in a sense extrapolative expectations: if the stock market price has been rising, investors expect it to keep rising; and if it has been falling, they expect it to keep falling, that is,

$$\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma} \frac{P_t}{P_{t-1}} > 1 \Rightarrow e_t^k > 0 \quad \forall k, t$$

where the same holds for reversed inequalities.

As a result, types' beliefs tend to comove and so either both experience and inexperienced agents believe stock price to increase or to decrease. Although this same expected qualitative change holds for most periods, the existence of an upper bound of beliefs implies that there could be times in which agents revise beliefs in opposite directions. That's because, depending on the difference between types' beliefs when PD ratio is approaching its upper limit, the price drop initiated by β_t^h is not enough to make realized price growth to be below β_t^l, resulting in $e_t^l > 0$ and L-type increasing its belief. In general, the comovement in beliefs holds as long as they are not sufficiently close to their upper bounds $\beta_t^k << \beta^U$ for $k = h, l$.

Regardless of its importance on shaping asset price behavior, the aforementioned case depends on a fine set of parameter values and beliefs range, and I abstein from it for now. For clarity, I impose the following assumption:

All agents, indepedently of their type, change their belief in the same direction. That is, $e_{t-1}^h > 0$ if anf only if $e_{t-1}^l > 0$ $\forall t$. Hence, it must be that $\Delta\beta_t^h > 0$ if and only if $\Delta\beta_t^l > 0$ $\forall t$. The same holds for reversed inequalities.

Forecast Errors

Firstly, intial beliefs updating equations (25) imply that agents revise $\beta_t^k, \forall k, t$ in the same direction as the last forecast error: beliefs increase (decrease) if investors underpredict (overpredict) risk-adjusted asset price growth. To infer about the magnitude of changes in beliefs one needs to compare realized and subjective price growth[11]: $\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\frac{P_t}{P_{t-1}}\times E_t^k\left\{\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\frac{P_t}{P_{t-1}}\right\}\equiv\beta_t^k$.

Price equilibrium equation (20) shows that, for any equilibrium weight λ_t^*, a higher (lower) $\beta_t^k, k=h,l$ delivers a higher (lower) realized PD ratio. In absence of further disturbances, this increases (decreases) risk-adjusted price growth. However, realized price growth will always strongly exceed the initial implied expected price growth. That's because actual risk-adjusted price growth is a steeper function of beliefs and its subjective expectations: when there is an increase in β_t^k, equilibrium price growth increase by more than 1 unit; the reverse holds for when subjective expectations decrease. This means that $|e_t^k|>0$.[12]

To be precise, when both agents hold the RE belief it is true that their forecast errors will be zero. Since the focus of this paper is to consider deviations from rational expectations, I assume this scenario has probability zero of happening.

Forecast error across types differ. By the characterizion of experience and inexperienced agents (see equations (22)-(25)), the latter has consistently more extrapolative beliefs and thus exhibits greater errors. Formally.[13]

Proposition 1. *At any period, the forecast error of inexperienced investors (H) have greater magnitude when compared to experienced's (L) error: $|e_t^h|>|e_t^l|$. Since $g^h>g^l$, it holds then that*

$$|g^h e_{t-1}^h|>|g^l e_{t-1}^l| \quad \forall t \tag{28}$$

In other words, difference in type's reactions is suffciently high so that the absolute change in beliefs of inexperienced agents is greater than the absolute change of experienced agents at every period:

$$|\Delta\beta_t^h|>|\Delta\beta_t^l| \quad \forall t \tag{29}$$

Combining the previous Proposition 1 and Assumption 5.1, it is clear that agents agree in which direction they will revise their beliefs to but disagree in the magnitude of this change. The overall evolution of types' beliefs β_t^h, β_t^l is such that, at every period, change in beliefs are qualitatively the same but quantitatively different across types.

[11]I will use the term price growth meaning risk-adjusted price growth for simplicity.
[12]Proof in Appendix B.
[13]Proof in Appendix B

5.2 Second-Stage: Social Interaction

After intial beliefs β_t^h, β_t^l are set, agents communicate with their neighbors and formulate their end-of-period beliefs $\tilde{\beta}_t^i = \lambda^i \beta_t^h + (1-\lambda)^i \beta_t^l$. Equilibrium price is then determined by the agent(s) with the highest end-of-period belief $\tilde{\beta}_t^i$, also referred to as the most optmistic agent. Embodied in $\tilde{\beta}_t^o$ are equilibrium weights on type-specific beliefs β_t^h, β_t^l denoted as $\lambda_t^*, 1-\lambda_t^*$, respectively. Therefore, equilibrium belief is given by

$$\lambda_t^* = \begin{cases} \max_i \lambda^i & \text{if} \quad \beta_t^h > \beta_t^l \\ \min_i \lambda^i & \text{if} \quad \beta_t^h < \beta_t^l \end{cases} \quad \text{equilibrium weight} \tag{30}$$

$$\tilde{\beta}_t^o = \lambda_t^* \beta_t^h + (1-\lambda_t^*)\beta_t^l \quad \text{equilibrium belief} \tag{31}$$

Notice that even though λ_t^* changes over time, λ_i's are fixed. This is because the network G is assumed to be exogenous and fixed. To determine equilibrium one must know not only agents' type but their location and neighborhood in the social network. Besides agents' degree of connectivity, the identity of one's friends matter for pinning down realized PD ratio.

Combining equilibrium equations (20), (30), (31) and belief updating equations (25), realized risk-adjusted price growth is expressed as

$$\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma} \frac{P_t}{P_{t-1}} = (a\epsilon_t^c)^{-\gamma}\left(a + \frac{a\delta\Delta\tilde{\beta}_t^o}{1-\delta\tilde{\beta}_t^o}\right)\epsilon_t^d \tag{32}$$

6 Price Dynamics

Together, the existence of a maximum for the PD ratio and beliefs are going to be key in giving rise to booms and busts in stock market asset price. Realized price and beliefs are consistently reinforcing each other and this constitutes a self-fulfilling mechanism that magnifies asset price changes: if stock market price goes up, this price increase feeds into investors' expectations about future price changes, which leads them to push the current price up even higher. This upward bid then makes realized asset price to increase further, resulting again in investors pushing the current price still higher, and so on.

Consider a situation in which agents become optimistic, meaning they increase their price growth expectations. This increase in expectations leads to an increase in the PD ratio, and consequently, realized price growth is greater and exceeds its initial expectations. The beliefs updating equations (25) then implies further upward revisions in price growth expectations and thus asset price increases further. In abscence of any fundamental shocks, this process leads to a sustained asset price boom in which the PD ratio and risk-adjusted price expectations jointly move upward. The price boom comes to an end when equilibrium PD function (20) gets closer to its maximum. At this point, realized price growth fails to fulfill agents' expectations which in turn leads to a reversal in the evolution of beliefs. Agents start to decrease their beliefs and the opposite dynamics above is set in motion.

Hence, an upper pound in the PD ratio implicates the asset price boom must come to an end. When this happen, price growth is compared to agents beliefs and such inconsistency will shoot $\beta_t^k, k = h, l$ a very low value. Booms and busts are detailed discussed in Section 66.2.

I examine the asset price behavior from three prespectives: PD ratio volatility, booms and busts dynamics and agents' disagreement. In order to focus in the extent of the causality between PD ratio fluctuations and agents' behavior, I abstein from major fluctations on the dividend process. That's to say that, even though it is true that a very negative/positive dividend realization could cause a significant decrease/increase in asset price, I rule out this scenario.

6.1 Price-Dividend Volatility

It is straightforward to see that changes in how agents perceive the stock market contributes to PD volatility. From equation (32), I have

$$Var\left(\ln \frac{P_t}{P_{t-1}}\right) \approx Var\left(\ln \frac{1-\delta\tilde{\beta}_{t-1}^o}{1-\delta\tilde{\beta}_t^o}\right) + \ln \varepsilon_t^d \tag{33}$$

Also, for a given λ_t^*, equilibrium belief volatility is given by

$$Var(\tilde{\beta}_t^o) = (\lambda_t^*)^2 Var(\beta_t^h) + (1-\lambda_t^*)^2 Var(\beta_t^l) \tag{34}$$

Thus, changes in type-beliefs β^h, β^l results in higher volatility of equilibium beliefs $\tilde{\beta}^o$, what contributes to greater price growth variation and thus, to higher PD ratio volatility.

6.2 Booms and Busts

I now show that a PD ratio boom and busts dynamics holds in the present framework emerging from agents optimally learning about equilibrium price process and social interaction. To begin with, suppose at time t investors expect a positive risk-adjusted price growth, that is $e^k_{t-1} > 0, k = h, l$. Then it holds that

$$\beta^k_t = \beta^k_{t-1} + g^i \left[\left(\frac{C_{t-1}}{C_{t-2}} \right)^{-\gamma} \frac{P_{t-1}}{P_{t-2}} - \beta^k_{t-1} \right] \Rightarrow \beta^k_t > \beta^k_{t-1} \quad \forall k = h, l$$

Both types' private belief increases and since experienced agents are more reactive to price growth observations $\beta^k_t > \beta^l_t$. Communication results in end-of-period beliefs $\tilde{\beta}^i_t$ $\forall i \in N$ and equilibrium is determined by $\lambda^*_t = \max_i \lambda^i$. Hence, realized PD ratio and price growth are, respectively

$$\frac{P_t}{D_t} = \frac{\delta \beta^{RE}}{1 - \delta \tilde{\beta}^o_t} = \frac{\delta \beta^{RE}}{1 - \delta [\lambda^*_t \beta^h_t + (1 - \lambda^*_t) \beta^l_t]}$$

$$\frac{P_t}{P_{t-1}} = [a^{1-\gamma} (\epsilon^c_t)^{-\gamma} \epsilon^d_t] \left(\frac{1 - \delta \tilde{\beta}^o_{t-1}}{1 - \delta \tilde{\beta}^o_t} \right)$$

Since both β^h_t, β^l_t are greater than the respective last period beliefs, $P_t > P_{t-1}$ no matter the identity of the most optimistic agent in the the previous period $\tilde{\beta}^o_{t-1}$. At $t+1$, $e^k_t > 0$ $\forall k$, investors revise their beliefs upwards and asset price continues to increase. In the aftermath, $\beta^h_{t+s} > \beta^l_{t+s}$ for all $s \geq 1$, equilibrium is pinned down by $\lambda^*_{t+s} = \max_i \lambda^i$, and realized price growth is greater than one, increasing and exceeds subjective expectations $e^k_{t-s} > 0$. Hence, a boom dynamics is in place in which future expectations and asset price continues to increase, and the most optimistic agent is the one who has more inexperiend friends.

Due to its greater reactivenes, H-type will be the ones firstly disappointed by price realizations. As the PD ratio gets closer to its maximum value, equilibrium price growth increase will fail to fulfill β^h_t magnitude. At this point, H-expectation is bigger than realized price growth which means they start revising beliefs downwards. Consequently, price decreases and its drop is high enough to also make L-agents to revise their belief downwards.[14] A bust episode is in motion then.

The resulting price behavior is similar to what Adam et al. (2016) study but price

[14]This follows from Assumption 5.1

realizations themselves differ since now communication is crucial for price setting and both types influence equilibrium. Nonetheless, the main mechanism behind the evolution of subjective risk-adjuted price growth beliefs is the same[15] As they show, the individual learning problem (21)-(26) results in beliefs β_t^h, β_t^l to stochastically oscilate the RE value and their upper bound β^U cannot be an absorbing point. This renders a momentum and mean-reversion behavior into the PD ratio. I refer to them for the details and analytical proofs of theses results.

The main difference between booms and busts is which type dictates equilibrium price. Booms are characterized by inexperienced agents being more influential whereas in busts experienced agents' beliefs are the main driving source. These episodes impart a high volatily of the PD ratio, in line with standard asset price evidences that motivate this paper.

6.3 Recovery Periods

In the periods between booms and busts events and in asbecence of further disturbances, stock price return to its mean deterministic value and oscilate much less around it. These recovery periods are marked by higher difference in private beliefs β_t^h, β_t^l such that $\beta_t^l > \beta_t^h$.

To see why that's the case, consider the bust following a boom. Stock price is continuosly decreasing and both types are revising beliefs downwards. H-type changes belief more aggresively and so β_t^h is smaller than L-type beliefs. Even when price start to increase again this continues to be true because of how further down inexperienced investors have pushed their expectations. It takes some periods of underpredictions - that is, $e_t^k > 0$ $k = h, l$ - for β_t^h to catch up with β_t^l.[16] When that happens, the optimistic behavior of investors has lead to such increase in asset price that a boom phase is in motion again.

The described stock price behavior implies that the speculative behavior of investors is an important mechanism behind it. The expectation of more speculative investors (here the H-type) influence others in the market and leads to a more rapidly increase/decrease in asset price, since beliefs are mutually reinforcing each other and realized price growth. Communication then results in they sharing similar expectations in booms/-busts episodes. When asset price returns to its mean behavior, beliefs differ more and the

[15] This follows because both model, ours and Adam et al. (2016)'s, share pratically the same individual learning scheme (21)-(26).

[16] Proof in Appendix C

experienced investors (L-type) are more influential in determining equilibrium price.

6.4 Duration of Booms and Busts

During a boom or a bust λ_t^* remains unchanged.[17] Let $\lambda_t^* = \lambda_{t-1}^* \equiv \lambda^*$ The effect of the social variable on risk-adjusted price growth is:

$$\frac{\partial\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\frac{P_t}{P_{t-1}}}{\partial\lambda^*} = \delta a^{1-\gamma}\rho_t\left[\frac{(\beta_t^h - \beta_t^l)(1-\tilde{\beta}_{t-1}^o) - (\beta_{t-1}^h - \beta_{t-1}^l)(1-\tilde{\beta}_t^o)}{(1-\tilde{\beta}_t^o)^2}\right] \tag{35}$$

The above is positive whenever asset price and agents' expectations are increasing., and negative in the opposite case. During a boom phase, when price keeps getting bigger, a higher λ^* results in a higher realized price growth. Consequently, price grows faster the higher is λ^* and the duration of the boom is smaller. In a bust episode the same result holds but a higher λ^* implies a lower realized price growth which implies price falls faster and the bust duration is also smaller.

Agents' social network is determinant of the duration of booms and busts. The greater the change in equilibrium belief between two periods the more intense (smaller duration) will be a boom episode since price growth realizations will be greater, and, consequently, the PD ratio function will get closer to its maximum quicker. Intuitively, when agents are heavily influenced by their more speculative friends - that is, when λ^i are relatively big - they incorporate more of the latter expectations into their end-of-periods beliefs $\tilde{\beta}^i$.

These results also imply that the magnitude of boom/busts, that is the volatility and maximum value of realized PD ratio, are also influenced by λ^*. A social network that results in a higher $\max_i \lambda^i$ when price is increasing will have shorter boom and bust durations, greater PD ratio volatility and higher PD ratio maximum.

Finally, the change in equilibrium beliefs $\Delta\tilde{\beta}_t^o$ is also a function of the communication varible. During a boom or bust episode,it holds that

$$\frac{\partial\Delta\tilde{\beta}_t^o}{\partial\lambda_t^*} = (\beta_t^h - \beta_{t-1}^h) - (\beta_t^h - \beta_{t-1}^l)\begin{cases} > 0 & \text{if} \quad e_t^k > 0 \\ < 0 & e_t^k < 0 \text{if} \end{cases} \tag{36}$$

Hence, during a boom (bust) equilibrium belief change is is greater (smaller) the bigger is λ_t^*, and consequently the higher is realized PD ratio.

[17] See equation 27. Booms are characterize by $\beta^h > \beta_t^l$ and so $\lambda_t^* = \max_i \lambda^i$ for all in its duration. The reverse holds for a bust:$\beta^h < \beta_t^l$ and so $\lambda_t^* = \min_i \lambda^i$

Keep in mind that λ^i gives how agents are proportionally connected to inexperienced investors and λ_t^* is simple the maximum or minimum λ^i depending whether price is increasing or not. A higher λ^i means that an agent has relatively more H-friends. Since in booms episodes H-type private belief β^h is always higher than the L-type one, a higher λ^i (and consequently λ_t^*) results in a higher weight to those greater belief β_t^h and so equilibrium price will be greater. However, if λ_t^* is relatively small, a greater weight in equilibrium price is given to the smaller belief β_t^l and that's why realized PD ratio will be comparatively smaller. The same reverse reasoning holds for when price growth is below one.

7 Equilibrium and Network Topology

As discussed above, agents' degree and the network strucure are determinant of equilibrium prices and the characterization of booms and busts. To study price dynamics, I need to know for any agent at any point of time his type-belief $\beta_t^{i,k}$ and his social structure λ^i. To know λ^i, all agents' type $i_k \; \forall i \in N$, their neighboors and these later identities $j_k \in N_i$ must be known. Therefore define the state of each agent as $x_i = (i_k, D_i^h, D_i^l)$ where i's degree is $D_i = D_{ih} + D_{il}$. Let the total degree of i be the actual number of friends of he has, $\bar{D}_i = D_i - 1$. Given individual states I can infer the stock market equilibrium outcome and price behavior. A natural outcome of our model is then an algorithm to compute agents' set of final beliefs and asset price outcomes (risk-adjusted price growth, price-dividend ratio, stock return) for a given parametrization.

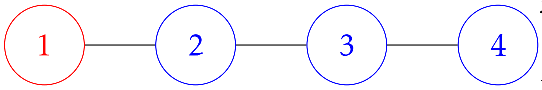
Next I present some simple examples of network graphs and how much information about asset price behavior I can get out of their structure.

7.1 Some Useful Examples

Consider four different networks structures: a tree, a complete network, a star and a circle. Apart from this, the economy is exactly the same in all of them, namely, equal shares of H and L agents.

Graphicaly, H nodes are colored in red and L nodes in blue. I report network mean degree, individual state, nodes' communication variable λ^i and equilibrium belief equation $\tilde{\beta}_t^o$.

Tree



$$E(d_i) = \frac{3}{2}$$

$$x_1 = (H,1,1), \quad x_2 = (L,1,3), \quad x_3 = (L,0,3), \quad x_4 = (L,0,3)$$

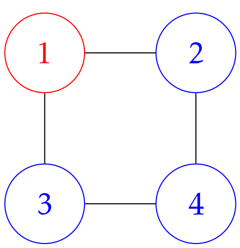
$$\lambda^1 = \frac{1}{2}, \quad \lambda^2 = \frac{1}{3}, \quad \lambda^3 = 0, \quad \lambda^4 = 0$$

$$e_{t-1} \begin{cases} > 0 & \lambda_t^* = \lambda^1 \rightarrow \tilde{\beta}_t^o = \tilde{\beta}_t^1 = \frac{\beta_t^h + \beta_t^l}{2} \\ < 0 & \lambda_t^* = \lambda^3 = \lambda^4 \rightarrow \tilde{\beta}_t^o = \tilde{\beta}_t^3 = \tilde{\beta}_t^4 = \beta_t^l \end{cases}$$

In a tree network, there is always two agents with just one link (those at the end poitns) and all the reamining nodes have total degree equal to 2. This is a network with a few connections. In general, for any network size N and any social structure (types and neighboords), there will be only three possible values for end-point nodes' weights $\lambda^i = \{0, 1/2, 1\}$, also three possible λ^i for nodes with degree 2: for H-type nodes $\lambda^i = \{1/3, 2/3, 1\}$ and for for L-type nodes $\lambda^i = \{0, 1/3, 2/3\}$. Hence end-of-period beliefs $\tilde{\beta}_t^i$ do not vary greatly across agents.

In the above example, equilibrium price is given either by agent 1 - when asset price is increasing - or by agent 4 - for decreasing price.

Circle



$$E(d_i) = 2$$

$$x_1 = (H,1,2), \quad x_2 = (L,1,2), \quad x_3 = (L,1,2), \quad x_4 = (L,0,3)$$

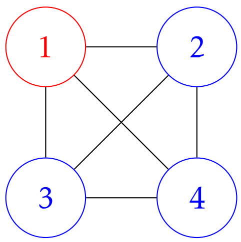
$$\lambda^1 = \frac{1}{3}, \quad \lambda^2 = \frac{1}{3}, \quad \lambda^3 = \frac{1}{3}, \quad \lambda^4 = 0$$

$$e_{t-1} \begin{cases} > 0 & \lambda_t^* = \lambda^1 = \lambda^2 = \lambda^3 \rightarrow \tilde{\beta}_t^o = \tilde{\beta}_t^1 = \tilde{\beta}_t^2 = \tilde{\beta}_t^3 = \frac{\beta_t^h + 2\beta_t^l}{3} \\ < 0 & \lambda_t^* = \lambda^4 \rightarrow \tilde{\beta}_t^o = \tilde{\beta}_t^4 = \beta_t^l \end{cases}$$

In a circle network, all agents have the same total degree of order 2 and this is also a network with a few connections. The set of values of λ^i is limited regarless the size of the network: for H-type nodes $\lambda^i = \{1/3, 2/3, 1\}$ and for for L-type nodes $\lambda^i = \{0, 1/3, 2/3\}$. Hence end-of-period beliefs $\tilde{\beta}_t^i$ not dot vary greatly across agents.

In the example, it is clear that most agents sharethe same communication structure λ^i, regardless of their types.

Complete

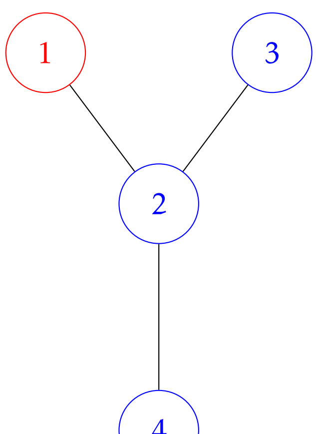


$$E(d_i) = 3$$
$$x_1 = (H,1,3), \quad x_2 = (L,1,3), \quad x_3 = (L,1,3), \quad x_4 = (L,1,3)$$
$$\lambda^1 = \frac{1}{4}, \quad \lambda^2 = \frac{1}{4}, \quad \lambda^3 = \frac{1}{4}, \quad \lambda^4 = \frac{1}{4}$$
$$\lambda = \lambda^1 = \lambda^2 = \lambda^3 = \lambda^4$$
$$\lambda_t^* = \lambda \rightarrow \tilde{\beta}_t^o = \tilde{\beta}_t = \frac{\beta_t^h + 3\beta_t^l}{4} \quad \forall e_{t-1}, t$$

The complete graph is characterized by all agents beign connected to one another. Because of this, they share the same degree of order $N-1$. In fact, λ^i is the same across agents and periods. Hence there is just one possible constant value for equilibrium weight regardless of price behavior. This implies that end-of-period beliefs $\tilde{\beta}_t^i$ are the same for all investors and I can think of this as a case of an representative agent with belief characterized by λ^*.

The graph above illustrates the aforementioned characteristics: the degree of all nodes is 3 and $\lambda^* = 1/4$.

Star



$$E(d_i) = \frac{3}{2}$$
$$x_1 = (H,1,1), \quad x_2 = (L,1,3), \quad x_3 = (L,0,2), \quad x_4 = (L,0,3)$$
$$\lambda^1 = \frac{1}{2}, \quad \lambda^2 = \frac{1}{4}, \quad \lambda^3 = 0, \quad \lambda^4 = 0$$
$$e_{t-1} \begin{cases} > 0 & \lambda_t^* = \lambda^1 \rightarrow \tilde{\beta}_t^o = \tilde{\beta}_t^1 = \frac{\beta_t^h + \beta_t^l}{2} \\ < 0 & \lambda_t^* = \lambda^3 = \lambda^4 \rightarrow \tilde{\beta}_t^o = \tilde{\beta}_t^3 = \tilde{\beta}_t^4 = \beta_t^l \end{cases}$$

In a star network, all nodes have just one link except for the center which has total degree of $(N-1)$. Thus, there are infinetly-many possible λ_t^* realizations since the center's weight will depend on the size and strcuture of the network. For the end-nodes, there are just three possible values for $\lambda^i = \{0, 1/2, 1\}$ and depending on the center's type this set of values actually gets smaller: if the center is of H-type, $\lambda^i = \{1/2, 1\}$ and if it is of

L-type $\lambda^i = \{0, 1/2\}$. Even though the center has the most connections, it will never be the unique optimistic agent: eitheir its λ^i is not a maximum/minimum or it is equal to at least another node's λ^i.

The example shows that since the center is an experienced agent, equilibrium is always given by the low-degree (equal to 1) nodes: when price is increasing, λ^* is from agent 1 , and when price is decreasing λ^* is given by agents 3 and 4.

7.2 On a Generic Network

Comparing events of a continuum increase and decrease of stock market price under the networks examplified above enable us to draw some conclusion of price dynamics as a function of the network structure.[18] For a given proportion of H and L agents in the economy, a more connected network results in lower disagreement - the dispersion of end-of-period beliefs $\tilde{\beta}_t^i$ across agents - and lower PD ratio volatility. In turn, in such social structure boom and bust episodes exhibits less strength - in terms of the magnitude of price increase - and last for longer periods.

That's because high connectivity increases the probability of different types sharing more linkages. Also, agent's type are less relevant the higher the mean degree. In the limit of a fully connected network, equilibrium communication weight λ_t^* is constant and the same across agents. Equilibrium price is then determined solely by the proportion of H and L nodes in the economy.

The following proposition summarizes this discussion.

Proposition 2. *Consider a network (G, N) with N_h inexperienced investors and N_l experienced ones. Denote the individual degree of a type $k = \{h, l\}$ node as D_{ik}. The following are true:*

- *in the case G is fully connected, $\lambda^i = \lambda_t^* = \frac{N_h}{N}$ for all periods t, and all agents regardless of their type.*
- *If $E(D_{ih}) > N_h - 1$, then $Prob(\lambda_t^* = 1) = 0$. On the other hand, if $E(D_{ih}) \leq N_h - 1$, then $Prob(\lambda_t^* = 1) = \frac{1}{(N_h-1)N_l}$.*
- *if $E(D_{il}) > N_l - 1$, then $Prob(\lambda_t^* = 0) = 1$. On the other hand, if $E(D_{il}) \leq N_l - 1$, then $Prob(\lambda_t^* = 0) = \frac{1}{(N_l-1)N_h}$.*
- *as long as $D_{ik} > N_k - 1$, $\lambda^i \neq \{0, 1\}$ $\forall i \in N$. Moreover, $\lambda^{ih} \in [\frac{1}{N_h}, 1)$ and $\lambda^{il} \in (0, \frac{1}{N_h}]$.*

[18] Refer to appendix D for a detailed comparison of these examples.

As long as there are no agent only connect with others of its type, $\lambda^i \neq \{0,1\}\ \forall i$ and so high network connectivity results in lower equilibrium weight λ_t^* at any period such that $\lambda_t^* \in \{0,1\}$. In such case, investors of different types are more connected and thus share more beliefs. This is captured by weighting beliefs similarly, that is a low λ^i.

8 Simulation

To evaluate the quantitative potential of the model on replicating the relevant empirical asset price features, I simulate the model for different network structures.

I follow the calibration (see Table 1) and numerical algorithm of Adam et al. (2016).[19] I also take advantage of their data sources to evalute the match of empirical findings (Table 2) and the model outcomes.

parameter	δ	γ	a	s_d	s_c	$\rho_{c,d}$	β^{RE}	PD_{max}	g_h	g_l
value	1.0	5	1.001	0.0245	$\frac{s_d}{7}$	0.02	0.9961	500	0.0091	0.0061

Table 1: Simulation Calibration
For types' gain, I consider a 20% variation of the gain used in AMN.

Fact	Data
PD ratio Mean	123.91
PD ratio Variance	62.43
Stock return	2.25
Stock return Std. Dev	11.44
Bond return	0.15
Risk Premium	2.10
Dividend Growth Mean	0.41
Dividend Growth Std. Dev.	2.88

Table 2: U.S. Asset Pricing Facts, 1927:2 to 2012:2
Source: Adam et al. (2016)

I consider seven types of network structures by varying the social variable λ^i, given the assumption that all social connections are known by the econometritian. Since I know that only the maximum and minimum λ^i matter for pinning down equilibrium, it is enough to impose two values for λ^i to analyze stock market behavior. Table 3 reports

[19] Details on the simulation algorithm can be found in Appendix E

the quantitative outcomes. The thinking exercise is to imagine the same economy - as described in the previous sections - with many individuals who can be rewired differently. The set of possible connections is captured by λ's values, so in a sense λ is a parameter and choosing its possible values means taking a stand on how the social connections look like. Throughout all the simulations, the economy is exactly the same - equal exogenous shocks, individual gains and aggregate proportion of types - the only parameter changing is λ.

Firstly, figures 1, 2 and 3 show realizations of time-series outcomes of our variables of interest generated from simulating the calibrated model specification of $\lambda_t^* = \{\frac{1}{2}, 1\}$. I display the evolution of the PD ratio, realized price growth, types' risk-adjusted price growth beliefs, equilbrium weight and belief. The simulated time series for the PD ratio reproduce booms and busts similar those I observe in the data, reported by Adam et al. (2016) and Adam et al. (2017) for example. Figure 1 display the fact that, even though dividend follow an stochastic process, its growth rate oscilates around its mean value of one which means a constant dividend growth process.[20] Comparing equilibrium belief and dividend fluctuations I see that the one responsible for the boom and busts are the former: time series of $\tilde{\beta}_t^o$ and PD_t move together.

Figure 2 displays some characteristics of the dynamics of our model: inexperienced agents are more reactive and have more volatile beliefs; and outside of booms and busts episodes the L-type agents is more optimistic. By looking at Figure 3 I see that experienced investors are the most influencial across the simulated periods. H-type beliefs matter the most for equilibrium under boom and busts. I do not report individual end-of-period beliefs $\tilde{\beta}_t^i$ but it is known they are bounded by β_t^h and β_t^l, and so they would be graphically located in between these time series in Figure 2.

The second exercise is to compare outcomes under different networks. The first four rows of Table 3 picture scenarios in which λ^* oscilates between two values and the highest (lowest) one holds when price is continously increasing (decreasing). For example, $\lambda = \{\frac{1}{2}, 1\}$ implies that $\tilde{\beta}_t^o = \beta_t^h$ if $\beta_t^h > \beta_t^l$, that is during a boom, and $\tilde{\beta}_t^o = \frac{\beta_t^h + \beta_t^l}{2}$ if $\beta_t^h < \beta_t^l$, that is, during the bust. Consider first the cases where λ varies. Not surprisinly, the social structure exhibiting greater PD ratio volatility is the one assigning higher weight to β^h at any period (first and fourth rows). These same specifications delievers a lower PD ratio mean. Booms have shorter duration and recovery periods are greater for higher values of λ.

[20]This is clear by looking at the model dividend growth process speficiation (1)

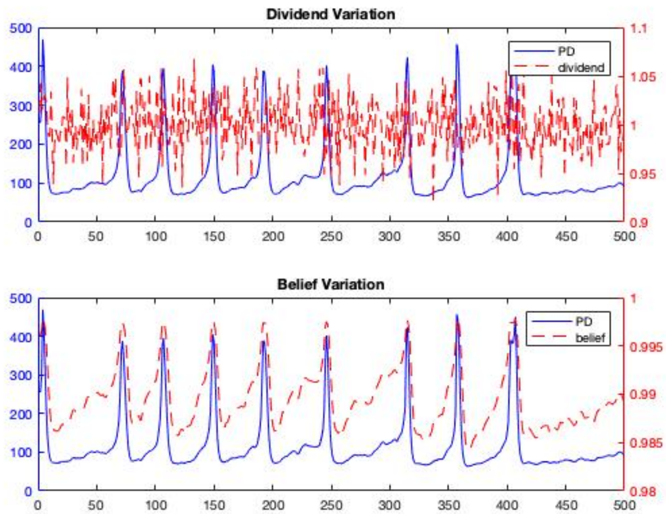


Figure 1: PD Ratio Volatility Sources
Figure compares model resulting processes for PD ratio, dividend growth and equilibrium belief under the specification of $\lambda = \{\frac{1}{2}, 1\}$.
All x-axis are time periods in quarters.

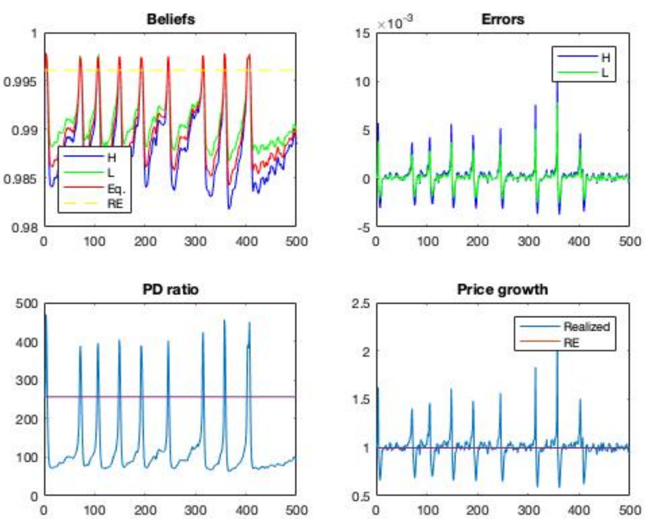


Figure 2: Simulation Outcome
Figure shows model resulting processes for types and equiblrium beliefs, types' forecasting errors, PD ratio and realized risk-adjusted price growth (in this order) under the specification of$\lambda = \{\frac{1}{2}, 1\}$. All x-axis are time periods in quarters.

For the cases when λ is constant, PD ratio volatily increases significantly compared to the cases discussed above. Interestingly, the PD ratio is most volatile when both agent's types equally influece asset price and booms last the longest.

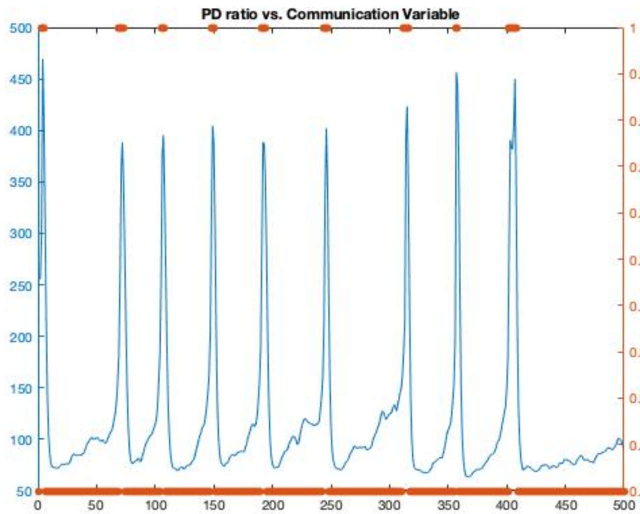


Figure 3: PD Ratio vs. Equilibrium Social Variable λ_t^*
Figure pictures the equlibrium λ_t^* at each PD ratio realization under the specification of $\lambda = \{\frac{1}{2}, 1\}$. The x-axis is time periods in quarters.

λ	PD_t	r_t^s	$r_t^s - r^b$	boom	recovery
$\{\frac{1}{2}, 1\}$	117.8340 (78.6245)	1.0161(0.1469)	1.0133 (0.1397)	3.7500	46.125
$\{0, 1\}$	132.9476 (70.4948)	1.0109 (0.1166)	1.0081 (0.1075)	4.7143	52.4286
$\{0, \frac{1}{2}\}$	140.0370 (73.9671)	1.0104 (0.1121)	1.0075 (0.1025)	5	44.8750
$\{\frac{1}{4}, \frac{2}{3}\}$	130.8282 (79.7531)	1.0126 (0.1236)	1.0098 (0.1150)	5	44.7500
0	139.0173 (73.5757)	1.0105 (0.1115)	1.0077 (0.1019)	5.1250	45.1250
$\frac{1}{2}$	125.1482 (85.4103)	1.0147 (0.1337)	1.0119 (0.1258)	5.5000	44.2500
1	106.4490 (82.6147)	1.0205 (0.1540)	1.0176 (0.1471)	4	45.7500

Table 3: Simulation Results
Risk-free interest rate is $r_t^b = 0.0048$ under all specifications. The implied dividend process have a mean dividend growth of 1.0012 and standard deviation of (0.0262). In brackets are the standard deviations of each varible. Boom and recovery variables are measured in quarters.

Comparing stock return across the different social structures I see that the major difference concerns its volatility. More volatile return coincides with more volatile PD ratio. There seems to be no evidence of neither higher PD ratio mean nor higher PD ratio volatility resulting in higher asset return. The resulting excess volatility of stock returns, defined as the much higher volatility of r_t^s compared to dividend growth volatility, matches the data. The equity premium outcome varies little across specifications and it is about half of its empirical mean value.

These results can be counterintuitive since one could expect that the more influential are inexperienced investors, what leads to a more volatile stock market, would require a higher equity premium to compensate for a higher risk. In fact, Barberis et al. (2015) find that, in a different framework where agents know the price process, the equity premium rises as the fraction of agents with more volatile beliefs about future asset price changes in the economy goes up. Under the baseline model of Adam et al. (2016), they are able to reproduce the empirical risk premium only at a sufficiently high level of risk aversion ($\gamma = 80$).[21]Thus, contrasting our result to the aforementioned ones suggests that a better understading of what are the roots of such observed high risk premium in the stock market is an important research avenue.

Notice that the dividend process in our framework has a lower mean value and variance, compared to the process implied by the data. Also, the simulated bond return is significantly smaller than its observed counterpart. The above suggests then the the main driving source of stock return outcome in the model is due to PD ratio behavior.[22]

Overall, the specifications replicate the empirical evidence of highly volatile PD ratio which can be found in Table 3. It is also in line with empirical PD ratio mean and excessive volatility of stock returns. Accounting for the existence of different types of investors seems to be an important feature of modeling stock market behavior.

9 Unkown Network Structure

The ongoing discussion has been focusing on what price dynamics would emerge for an exougenously given network. Some might argue that having the full knowledge of the underlying social connections of the stock market is overwhelming. Yet (partially) true, network theory comes in hand to overcome this obstacle and it enables the study of market outcomes when one does not know the full social structure. Instead of characterizing all existing edges among agents, I will assume that what is known are the probabilities of linkages between and across types. In this sense, I take a step back on the model framework and consider a more general setup.

The main idea behind random graph models is to suppose a random process is responsible for the formation of the links and then to randomly choose a network out of all the possible networks with such linkages, each network having an equal probability of being chosen. The properties of such random networks serve as a useful benchmark and provide some insights into the properties that some social and economic networks have.

[21] Adam et al. (2016) quantitative results can be found in the Appendix E

[22] To see this notice that I can write $r_t^s = \frac{1+PD_t}{PD_{t-1}} \frac{D_t}{D_{t-1}}$.

I analyze a variation of the well-known Erdos-Reny Random Graph Model (ERDdS and R&wi (1959))[23] with heterogenous links. But instead of modeling the network formation as a purely random process, I assume it results from a strategic behavior of agents. In our context, agents tend to be connected with those who share the same type. Hence, this simplified version of so-called Islands-Connections Model of Jackson and Wolinsky (1996) and our goal is to study the impact of homophily on stock market equilibrium.

Homophilly is the tendency to disproportionally stabilish connections with those having similar traits. It has been the objective under analysis the network literature. For example, Golub and Jackson (2012) study a model of friendship formation and the pattern of linkages that emerges from homophily. Golub and Jackson (2012) study homophily under the context of social learning and investage it effects on the convergence of consensus.

An Equal-Sized Two-Island Model

Under a similar framework as above, let agents be divided in two groups $k = h, l$ each with mass μ_k, such that $\mu_h + \mu_l = 1$. For simplicity, assume groups are of the same size with $\mu_k = \mu = 0.5$ for all $k = h, l$. The network formation is such that links within a type are more probable than links across types, and the probability of those across types does not depend on the specifics of the types in question. An agent only distinguishes between agents of his own type and agents of a different type same-type; and they are all symmetric in how they do this. In a sense, other individual characteristics do not matter for a connection to exist.

The probability that an individual of type k has an (undirected) link to an individual of a different type k' is given by $p_{kk'} = p_{out}, \quad \forall k \neq k'$ and the probability that a link is formed with an individual of the same type is given by $p_{kk} = p_{in}, \quad \forall k$. All links are formed independently.

A measure of homophily is to compare the difference between same and different linking probabilities to the average linking probability, with a normalization of dividing by the number of groups[24]:

[23]The Erdos-Reny model is the simplest random graph model in which edges are i.i.d. random varibales. All edges are independent and are formed with the same probability p. Hence, the distribtuion of all edges is a Binomial $Bin(n, p)$

[24]This homophily measure is based on the work of

$$p = \frac{p_{in} + p_{out}}{2} \tag{37}$$

$$\tilde{H} = \frac{p_{in} - p_{out}}{2p} \quad \in [0,1] \tag{38}$$

where $p_{in} \geq p_{out}$ If nodes only links to same-type nodes (so that $p_{out} = 0$), then $\tilde{H} = 1$ and if nodes do not pay attention to type when linking ($p_{in} = p_{out}$), then $\tilde{H} = 0.5$.

The proportion of agents' neighbors of a given type is then a function of network homophily. In turn, the social variables $\lambda_k, k = h, l$ are given by[25]

$$\lambda_h = \frac{p_{in}}{p_{in} + p_{out}} \quad \& \quad \lambda_l = \frac{p_{out}}{p_{in} + p_{out}} \tag{39}$$

The above can be written in terms of the homophily index $\tilde{H}$:

$$\lambda_h = \frac{1 + \tilde{H}}{2} \quad \rightarrow \frac{\partial \lambda_h}{\partial \tilde{H}} > 0 \tag{40}$$

$$\lambda_l = \frac{1 - \tilde{H}}{2} \quad \rightarrow \frac{\partial \lambda_l}{\partial \tilde{H}} < 0 \tag{41}$$

As homophily increases, communication variable λ_k increases for the H-type and decreases for the L-type.[26] Connections are mainly within groups, agents do not share their beliefs and the equilibrium is either determined by inexperienced investors in the case of a boom, or by experienced one, during a bust, that is, equilibrium weight tend to its extremum values: $\lambda^* \rightarrow 1$ if $e_t^k > 0$ and $\lambda^* \rightarrow 0$ if $e_t^k < 0$.

Proposition 3. *For any degree of homophily and any set of private beliefs β_t^h, β_t^l such that $\lambda_h \geq \lambda_l$, equilibrium belief $\tilde{\beta}_t^o$ is given by inexperienced investors network structure λ_h whenever asset price is increasing, and by λ_l when price is decreasing:*

$$\tilde{\beta}_t^o = \begin{cases} \lambda_h \beta_t^h + (1 - \lambda_h)\beta_t^l & \text{if } P_t > P_{t-1} \\ \lambda_l \beta_t^h + (1 - \lambda_l)\beta_t^l & \text{if } P_t < P_{t-1} \end{cases} \tag{42}$$

Homophily implication for stock market outcome easily follows then: PD ratio volatity and the frequency of boom/busts are positive related to $\tilde{H}$, whereas the duration of such

[25]Please refer to the Appendix for the derivation,

[26]This is a trivial result since $\lambda_h + \lambda_l$ must sum up to 1.

episodes decreases as $\tilde{H}$ increases. This means that economies with segregated groups tend to exhibit a more espculative behavior in the stock market.

In fact, I can characterize asset price equilibrium behavior as a function of homophily.

Theorem 1. *Consider the learning asset pricing model characterized by equations* (1),(13), (25), (26) *with an unknown network structure. In the context of an equal-sized two-type random graph network formation model, define the homophily index $\tilde{H}$ as the normalized difference between same and different linking probabilities to the avarage linking probability and let $\lambda^h = \frac{1+\tilde{H}}{2}, \lambda^l = 1 - \lambda_h$. For any set of linkages probabilities and any degree of homophily, the price-dividend equilibrium equation is given by*

$$PD_t = \frac{\beta^{RE}}{\lambda_h^{\phi_t} \lambda_l^{1-\phi_t} \beta_t^h + \lambda_h^{1-\phi_t} \lambda_l^{\phi_t} \beta_t^l} \qquad \forall t, \beta_t^h, \beta_t^l \tag{43}$$

where ϕ_t is an indicator variable given by $\phi_t = 1_{\{\beta_t^h - \beta_t^l > 0\}}$ and zero otherwise, and $\beta^{RE} \equiv a^{1-\gamma}\rho$.

10 Concluding Remarks

This paper presents a simple asset pricing model with agents who have heterogenous subjective expectations about risk-adjusted price growth and socially interact, by sharing beliefs, through an exogenous given network. I show that the equilibrium asset price is a function of agents' beliefs and network structure. The model gives rises to boom and busts in the price-dividend ratio and such episodes characteristics, such as the duration and PD ratio volatility, depend on how communication among agents takes place. A key insight highlighted by the present framework is that booms (busts) are more influenced by agents who hold more (less) speculative beliefs.

One limitation of our study is our lack of asset pricing data apart from the U.S. stock-market. I also do not estimate the social network and take it as pre-determined. A next step of this research is then to estimate a network structure and apply its properties to the model, for different stock-markets.

An interesting avenue for future research could be to endogeneize the upper bound on agents' beliefs that it is needed in our model to guarantee a finite equilibrium price.

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A RE as a special case

If the RE assumption holds, then all agents know exactly what price outcome will be for any given history of realizations of the exogenous processes (D^t, Y^t). In other words, $E_t^{ik} = E_t \; \forall t, i, k$ where $E_t(.)$ is the expectation of the 'true' stochastic price process. Agents then can interate forward on equation (9):

$$\begin{aligned}
P_t &= \delta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} P_{t+1}\right] + \delta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} D_{t+1}\right] \\
&= \delta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left\{\delta E_{t+1}\left[\left(\frac{C_{t+2}}{C_{t+1}}\right)^{-\gamma} P_{t+2}\right] + \delta E_{t+1}\left[\left(\frac{C_{t+2}}{C_{t+1}}\right)^{-\gamma} D_{t+2}\right]\right\}\right] + \delta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} D_{t+1}\right]... \\
&= E_t\left[\sum_{s=1}^{\infty} \delta^s \left(\frac{C_{t+s}}{C_t}\right)^{-\gamma} D_{t+s}\right] + \lim_{s\to\infty} E_t\left[\left(\frac{C_{t+s}}{C_t}\right)^{-\gamma} P_{t+s}\right]
\end{aligned}$$

Assuming a no-rational-bubble requirement, it is common knowledge that

$$\lim_{s\to\infty} E_t\left[\left(\frac{C_{t+s}}{C_t}\right)^{-\gamma} P_{t+s}\right] = 0$$

Hence, equilibrium asset price is equal to the expected discounted sum of dividends

$$\begin{aligned}
P_t &= E_t\left[\sum_{s=1}^{\infty} \delta^s \left(\frac{C_{t+s}}{C_t}\right)^{-\gamma} D_{t+s}\right] \\
&= E_t\left[\sum_{s=1}^{\infty} \delta^s \left(a^s \prod_{j=1}^{s} \varepsilon^c_{t+j}\right)^{-\gamma} D_t (a^s \prod_{j=1}^{s} \varepsilon^d_{t+j})\right] \\
&= D_t\left[\sum_{s=1}^{\infty} \delta^s (a^{1-\gamma})^s\right] \prod_{j=1}^{s} E_t\left(\varepsilon^c_{t+j}\right)^{-\gamma} E_t\left(\varepsilon^d_{t+j}\right) \\
&= D_t\left[\sum_{s=1}^{\infty} \delta^s (a^{1-\gamma})^s \rho^s\right] \\
&= \frac{\delta a^{1-\gamma}\rho}{1-\delta a^{1-\gamma}\rho} D_t
\end{aligned}$$

where I use definition (12), $\rho = E[(\varepsilon^c_{t+1})^{-\gamma}\varepsilon^d_{t+1}] \; \forall t$, and exogenous processes (1) and (2). Thus, defining $\beta^{RE} \equiv a^{1-\gamma}\rho$ I get the rational expectations equilibrium asset price in equation (15).

B Proposition - Inexperienced agents are more reactive

First, to see why forecast errors are different than zero at any time priod, look at risk-adjusted price growth and its subjective expectations as a function of β_t^k:

$$\frac{\partial\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\frac{P_t}{P_{t-1}}}{\partial\beta_t^k} = \beta^{RE}\delta\lambda_t^{k,*}\frac{(1-\delta\tilde{\beta}_{t-1}^o)}{(1-\delta\tilde{\beta}_t^o)^2} > 1 \tag{44}$$

$$\frac{\partial E_t^k\left\{\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\frac{P_t}{P_{t-1}}\right\}}{\partial\beta_t^k} = 1 \tag{45}$$

Clearly, a change β_t^k implies a one-to-one change in agents' beliefs (by definition) but results in a greater change for realized price growth. Hence, the later always exceeds its intial expectations.

Now, Proposition 1 easily follows from Assumption 5.1. To see this consider an arbitrary period t in which price is increasing. So it holds that $\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\frac{P_t}{P_{t-1}} > 1$ and $e_{t-1}^k > 0$. By types' belief updating equation (25), it must be that agents revise beliefs updward at $t+1$: $\Delta\beta_{t+1}^k > 0\ \forall k = h, l$.

Since $g^h e_t^h > g^l e_t^l$, it must be that $\Delta\beta_{t+1}^h > \Delta\beta_{t+1}^l > 0$. Consequently, $\beta_{t+1}^h - \beta_{t+1}^l > \beta_t^h - \beta_t^l > 0$ and thus it must be that $\beta_t^h > \beta_t^l$ and $\beta_{t+1}^h > \beta_{t+1}^l$.

On the other hand, consider a period when price is decreasing such that $\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\frac{P_t}{P_{t-1}} < 1$ and $e_{t-1}^k < 0$. Again, by equation (25) it must be that agents decrease their beliefs next period: $\Delta\beta_{t+1}^k < 0\ \forall k = h, l$.

Since $g^h e_t^h < g^l e_t^l < 0$, it must be that $\Delta\beta_{t+1}^h < \Delta\beta_{t+1}^l < 0$. That is, $\beta_{t+1}^h - \beta_{t+1}^l < \beta_t^h - \beta_t^l < 0$ and thus it must be that $\beta_t^h < \beta_t^l$ and $\beta_{t+1}^h < \beta_{t+1}^l$.

As pointed out, these results follow from the Kalman filter equation of each type. The gains are sufficiently different to guarantee that $|g^h e_t^h| > |g^l e_t^l|\ \forall t$. In fact, the difference in forecast errors $e_t^h - e_t^l\ \forall t$ is virtually small because, for both types, beliefs are just small deviations of the RE value (which is the same for both). Consequently, the range of difference in beliefs is also sufficiently small so that the comparison of each with realized price itself is virtually the same. The optimal Kalman gains then are what make H-type beliefs to be more reactive to the forecast error and so to guarantee $|\Delta\beta_t^h| > |\Delta\beta_t^l|$ ∀ t.

This discussion is also clear by looking at simulation results in Section 8 (Figure 2). Importantly, despite the fact that beliefs oscilate around the RE value, these small deviations are sufficient to render the fluctuation in the PD ratio I obtain from the model. Adam et al. (2016) discuss this result in details and they argue that it is a strength of the model, since just slight departures from rational expectations are enough to reproduce stylized asset pricing facts.

C Recovery Periods

Let t be the period after a sharp continuous decrease in asset price following the boom - that is, at the bottom of the bust - it holds that $e_{t-1}^h < e_{t-1}^l < 0$ and $\beta_t^h < \beta_t^l$. Price (and so the PD ratio) returns to its mean value. In absecence of further shocks, price growth is approximately constant which implies that agents find themselves too pessimistic and start revising beliefs upwards according to the updating equations (25):

$$\begin{aligned} \beta_{t+1}^h &= \beta_t^h + g^h e_t^h \\ \beta_{t+1}^l &= \beta_t^l + g^l e_t^l \end{aligned} \quad \text{such that} \quad e_t^k > 0 \quad k = h, l$$

Then $\Delta\beta_{t+1}^k > 0$ $\forall k = h, l$. By mean reversion of the PD ratio, $\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma} \approx 1$ during the recovery periods. Then, I can rewrite the above as

$$\begin{aligned} \beta_{t+1}^h &\approx \beta_t^h(1 - g^h) + g^h \\ \beta_{t+1}^l &\approx \beta_t^l(1 - g^l) + g^l \end{aligned}$$

Since, $\beta_t^l > \beta_t^h$ and $g^h > g^l$ it holds that $(1 - g^l) > (1 - g^h)$. Because I consider gains to be small (see equation (26)), it must be that $\beta_{t+1}^l > \beta_{t+1}^h$. And so, for all period during recovery in which the PD ratio is approximately constant, $\beta^l > \beta^h$. After beliefs have consistently been increasing, price starts to increase sharply and so a boom episode is in motion.

D Network Graphs examples

To the extent of comparison between the examples presented above, let private beliefs $\beta_0^h = \beta_0^l = \beta_0$ be the same across structures and assume dividend and consumption shocks are also equal across networks.

Consider first a price growth greater than one as initial condition so that $P_0 > P_{-1}$, $e_0 > 0$ and this constitues a boom episode. At $t = 1$, $\beta_1^k \quad k = l, h$ is the same for all networks and $\beta_1^h > \beta_1^l$. Price is increasing and so equilibrium belief is pinned down by the highest λ^i. Realized PD ratio is $\frac{P_1}{D_1} = \frac{\delta\beta^D}{1-\delta\tilde{\beta}_1^o}$ where $\tilde{\beta}_1^o = \lambda_{max}\beta_1^h + (1-\lambda_{max})\beta_1^l$. The maximum λ^i is greater under the tree/star structure, followed by the circle and the complete structures, respectively: $\lambda^*_{tree/star} > \lambda^*_{circle} > \lambda^*_{complete}$. Thus, equilibrium belief and realized price at $t = 1$ also follow this magnitude relation. That is

$$\tilde{\beta}_1^{o,tree/star} > \tilde{\beta}_1^{o,circle} > \tilde{\beta}_1^{o,complete}$$
$$\frac{P_1^{tree/star}}{D_1} > \frac{P_1^{circle}}{D_1} > \frac{P_1^{complete}}{D_1}$$

Since λ_t^* has a positive effect on price growth during a boom, the higher is the former the greater will be the forecast error $e_1^k = \left(\frac{C_1}{C_0}\right)^{-\gamma}\frac{P_1}{P_0} - \beta_1^k > 0$ for $k = h, l$. Thus $e^k_{1,tree/star} > e^k_{1,circle} > e^k_{1,complete} > 0$. In the next period $t = 2$, it holds then that $\beta_2^{k,tree/star} > \beta_2^{k,circle} > \beta_2^{k,complete}$ $\forall k$ with $\beta_2^h > \beta_2^l$. Equilibrium belief and price will follow the above relation across networks so that PD ratio is the highest in the tree/star graph and lowest in the complete structure.

These results holds for all periods whenever expected price growth is positive, and under this scenario I can conclude that:

- the lower the network mean degree (tree, star) the higher is realized price and equilibrium belief at any period;
- less connected networks (tree, star) have greater price volatility when compared to more connected ones (circle and complete graphs)
- higher network mean degree implies lower dispersion of agents' public beliefs $\tilde{\beta}_t^i$

The analysis is analogous for a bust episode but results are qualitatively different. Let initial conditions be of a negative realized and expected price growth $P_0 < P_{-1}$ and $e_0 < 0$. In the first period intital beliefs are the same across networks such that $\beta_1^h < \beta_1^l$. Price is determined by the lowest λ^i and so $\frac{P_1}{D_1} = \frac{\delta\beta^D}{1-\delta\tilde{\beta}_1^o}$ where $\tilde{\beta}_1^o = \lambda_{min}\beta_1^h + (1-\lambda_{min})\beta_1^l$. Equilibrium λ^* is lower in the tree/circle/star networks and higher in the complete graph and thus

$$\tilde{\beta}_1^{o,tree/star/circle} > \tilde{\beta}_1^{o,complete}$$
$$\frac{P_1^{tree/star/circle}}{D_1} > \frac{P_1^{complete}}{D_1}$$

Because λ_t^* has a negative effect on price growth when price is decreasing, the higher is λ_t^* the lower (or greater in absolute value) will be forescast errors $e_1^k = \left(\frac{C_1}{C_0}\right)^{-\gamma}\frac{P_1}{P_0} - \beta_1^k < 0$ for $k = h, l$ since $\left[\left(\frac{C_1}{C_0}\right)^{-\gamma}\frac{P_1}{P_0}\right]_{complete} < \left[\left(\frac{C_1}{C_0}\right)^{-\gamma}\frac{P_1}{P_0}\right]_{tree/star/circle} < \beta_1^k < 0$. Thus $e_{1,complete}^k < e_{1,tree/star/circle}^k < 0$. In the next period $t = 2$ then, it holds that $\beta_2^{k,tree/star/circle} > \beta_2^{k,complete}$ $\forall k$ with $\beta_2^h < \beta_2^l$. Equilibrium belief and realized price will follow the above relation across networks so that PD ratio decrease is greater under the complete network compared to the tree/star/circle graphs.

These results holds for all periods whenever price is expected to decrease and I can infer that:

- more connected networks have higher equilibrium social weight λ_t^* and thus exhibits lower realized price
- price decrease is greater the higher the mean degree and so it is PD volatility

Notice that even though the tree and star graphs have different structure, both share the same expected degree and consequently they face the same price dynamics. In this sense, individual nodes' degree is not as relevant as the overall degree of network connectivity.

E Details on the Simulation

Under the calibrated parameters (Table 1), I simulated the model 1000 times throughout 500 quarters.

Our simulation method is close to Adam et al. (2016). The main difference is in our specification of agents' subjective beliefs. As explained by them, to guarantee beliefs' bound holds at all periods and equilibrium price always exist a differentiable projection facility in belief updating equation is introduced. Specifically, equation (25) becomes

$$\beta_t^k = \omega\left(\beta_{t-1}^k + g^k\left[\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\frac{P_t}{P_{t-1}} - \beta_{t-1}^k\right]\right) \quad \forall k = h, l \quad \& \quad t > 0 \tag{46}$$

$$\text{where} \quad \omega(x) = \begin{cases} x & x \leq \beta^L \\ \left(\frac{x-\beta^L}{x+\beta^U-2\beta^L}\right)\beta^U + \left[1 - \left(\frac{x-\beta^L}{x+\beta^U-2\beta^L}\right)\right]\beta^L & x > \beta^L \end{cases} \tag{47}$$

where $\beta^L = \frac{1}{\delta} - 2(\frac{1}{\delta} - \beta^U)$ and β^U is such that the PD ratio doesn't exceed 500. $\omega(x)$ is a dampening function that applies only to few observations - when beliefs are close tho their upper bounds. The upper bound on the PD ratio is chosen such that it is higher but close to its maximum value found in the U.S. data (which is approximately 375).

Booms are calculated as the number of time periods the PD ratio stays above its RE value at each cycle. Recovery periods are calculated as the number of time periods the PD ratio stays below its RE value at each cycle.

Since one of our study motivation is Adam et al. (2016), I specify their main set of results in Table E.1. I are able to repoduce similar empirical fact as them, but our model exhibit greater PD ratio volatility and mean depending on the social strcuture. Moreover, AMN do not discuss the lenght of booms and busts, but by looking at their results I can infer that the present model is able to account for the duration of these episodes. I consider this a relevant result since it implies that the presence of different investors in the market and how they behave are also determinant of the length and magnitude of booms and busts.

Fact	AMN	
	$\gamma = 5$	$\gamma = 80$
PD ratio Mean	122.50	115.75
PD ratio Std. Dev.	67.75	71.15
Stock return	1.27	2.11
Stock return Std. Dev.	10.85	16.31
Bond return	0.39	0.11
Risk Premium	0.88	2.0
Dividend Growth Mean	0.00	0.16
Dividend Growth Std. Dev.	2.37	4.41

Table E.1: Adam et al. (2016) Estimation Outcomes
Source: Adam et al. (2016)

F Unkown Network Structure

Define the expected number of links between types k and k' as

$$Q_{kk'} = \mu_k \mu_{k'} p_{kk'} = \begin{cases} \mu^2 p_{out} & k \neq k' \\ \mu^2 p_{in} & k = k' \end{cases} \tag{48}$$

and the expected sum of degrees of nodes of type k as

$$d_k = \sum_{k'} Q_{kk'} = Q_{kk} + Q_{kk'} = \mu^2 (p_{in} + p_{out}) \tag{49}$$

By the definition then λ for each type is given by

$$\lambda_h = \frac{Q_{hh}}{d_h} = \frac{p_{in}}{p_{in} + p_{out}} \tag{50}$$

$$\lambda_l = \frac{Q_{lh}}{d_l} = \frac{p_{out}}{p_{in} + p_{out}} \tag{51}$$

Proposition

By definition $\lambda_h \geq \lambda_l$, due to homophily. Recall the equilibrium equations (30) and (31) for the model with a known network strucute. The propostion statement is simply the most-optimistic belief equation $\tilde{\beta}_t^o$ replacing the communication variable.

To make the point clear, notice that during an price increase $\beta_t^h \geq \beta_t^l$. Since $\lambda_h \geq \lambda_l$, it must be that the higher weight is given to the higher type-belief and so $\tilde{\beta}_t^o = \lambda_h \beta_t^h + (1 -$

$\lambda_h)\beta_t^l$ during a boom. Whereas during a bust, $\beta_t^h \leq \beta_t^l$ and because $\lambda_l < (1-\lambda_l)$, it must be that $\tilde{\beta}_t^o = \lambda_l \beta_t^h + (1-\lambda_l)\beta_t^l$.

Theorem

Look at equations (20), (30) and (31) along with the proposition above. Since $\lambda_h > \lambda_l$, it holds that

$$\phi_t = 1 \leftrightarrow \beta_t^h > \beta_t^l \Rightarrow \tilde{\beta}_t^o = \lambda_t^h \beta_t^h + (1-\lambda_t^h)\beta_t^l$$
$$\phi_t = 0 \leftrightarrow \beta_t^h < \beta_t^l \Rightarrow \tilde{\beta}_t^o = \lambda_t^l \beta_t^h + (1-\lambda_t^l)\beta_t^l$$

Writting price equilibrium equation (20) as a function of $\phi_t, \lambda_h, \lambda_l$ delivers the Theorem.