Bank Opacity and Deposit Rates

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December 11, 2025

Abstract

We propose a novel mechanism for why bank portfolios are opaque: banks choose opacity to secure cheaper long-term funding while trading off insolvency and illiquidity risks. We show that while opacity lowers deposit rates, it also leaves depositors with only noisy information about the bank's solvency, making them cautious about keeping their funds in the bank—particularly when interest rates are high. Counterintuitively, opacity raises bank profits even though it forces the bank to tolerate a high risk of illiquidity. As a result, in high-rate environments, banks have stronger incentives to adopt excessive opacity to further reduce deposit rates—at the cost of more frequent early failures.

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We are grateful to Philip Bond, Leonardo D'Amico, Douglas Diamond, Thierry Foucault, Martin Oehmke, Cecilia Parlatore, Anatoli Segura, Alireza Tahbaz-Salehi and Victoria Vanasco for useful comments.

1 Introduction

Banks are opaque by design, and opacity remains a deliberate and enduring feature of modern banking. Investors and depositors often have limited insight into the composition and riskiness of a bank's assets—especially during times of stress. The literature offers two opposing views on the role of opacity in the financial sector. One perspective sees opacity as a source of fragility, increasing insolvency risk and fueling runs, panic, and systemic crises. The other emphasizes its role in mitigating illiquidity risk by enabling the creation of safe, information-insensitive debt (Dang et al., 2017). This tension over whether opacity ultimately supports or undermines financial stability remains unresolved. More fundamentally, given that banks in practice face both insolvency and illiquidity risk, under what conditions would a bank strategically choose to be opaque?

We approach the question by developing a model in which a bank that raises deposits to finance long-term risky projects must manage a trade-off between two distinct sources of fragility: illiquidity and insolvency. The main takeaway is that, to reduce long-term funding costs and raise cheaper deposits, the bank may rationally prefer a higher risk of illiquidity. To this end, she uses opacity as a strategic tool—not to obscure information for its own sake, but to lower funding costs by optimally trading off short-run fragility against long-run profitability.

We formally model this trade-off by allowing the bank to choose both how to allocate the funds she raises from depositors across different projects (portfolio allocation) and whether the portfolio is transparent, opaque, or fully unobservable to depositors (portfolio composition). Depositors retain the option to withdraw early and receive a safe outside alternative. Under transparency, depositors observe the return of each project at an interim date; under opacity, they observe only partial information at the interim date; and under full unobservability, no information is revealed before maturity. The bank's choices jointly determine both the risk profile of her portfolio and the information available to depositors when deciding whether to withdraw.

In choosing her portfolio of projects, the bank primarily cares about two objectives: avoiding failure and minimizing the promised deposit rate so as to maximize profit conditional on survival. Since the bank is funded with deposits, she can fail in two ways: she may become illiquid if depositors withdraw early, or she may become insolvent at maturity if her portfolio return cannot cover the promised return on deposits. When the bank can shape both the distribution of her portfolio returns and the information available to depositors through her portfolio choices, a trade-off between these two forms of fragility naturally arises.

To understand the underlying mechanism, it is useful to note that if we fix the portfolio composition to transparent or unobservable, which we call benchmark cases, the bank faces no trade-off between illiquidity and insolvency. It is so because with these two potrfolio compositions, depositor's information is given and independent of the bank's portfolio allocation. In the transparent case, depositors observe interim returns perfectly, eliminating uncertainty about the bank's solvency at the withdrawal date. The bank's objective is thus to minimize illiquidity risk and the associated return on deposits. She typically chooses a highly diversified portfolio that increases the likelihood of repayment and induces depositors to accept a lower return on deposits. However, when depositors outside option is very appealing, she shifts toward a more concentrated portfolio to capture upside gains—engaging in risk shifting.

In the unobservable case, depositors receive no interim information, which makes their decision to withdraw perfectly predictable and eliminates illiquidity risk. As a result—much like in the transparent case—the bank is encouraged to diversify, since a more stable return distribution lowers insolvency risk and reduces the return on deposits. In both settings, portfolio allocation affects only the return distribution, not the information available to depositors. This stands in contrast to the opaque case, where the information available to depositors depends on how the bank strategically manages the risk profile of her portfolio.

An opaque portfolio refers to a portfolio that is neither fully transparent nor fully unobservable. In particular, the depositors learn about part of bank's portfolio but not all of it, and the bank controls the share of the portfolio that the depositors learn about. As such, in presence of opacity, bank's ability to shape depositor beliefs becomes central to how she manages insolvency and illiquidity risks. In particular, the bank's portfolio allocation plays a dual role: it determines both the return distribution and the information that depositor rely on when deciding whether to withdraw. This dual function sets up conflicting incentives. Greater diversification improves solvency and lowers the long-term return on deposits, but it also blurs the information depositors use, increasing the likelihood of early withdrawals. Conversely, a more concentrated portfolio sharpens depositor signals and reduces illiquidity risk, but raises funding costs and insolvency risk. As a result, the bank chooses a portfolio allocation that balances—rather than minimizes—either risk.

The key result is that opacity gives the bank a lever to substitute between insolvency risk and illiquidity risk—enabling her to accept more frequent early withdrawals in exchange for a lower promised deposit rate and likelihood of default. This trade-off makes opacity an optimal and deliberate choice. In equilibrium, opacity emerges as both a source of fragility—fueling runs and early liquidation—and a mechanism for creating safer debt by reducing funding cost of the bank. This is consistent with the evidence offered by Chen et al. (2022) who find that transparent US banks pay higher deposit rates. In particular, we find that in high interest rate environments, opaque banks pay a lower deposit rate compared to transparent ones. Put differently, our model suggests that in high interest rate environments, bank strategically manipulate their portfolio towards being excessively opaque in order or minimize the transmission of monetary policy.

The model has implications about how the level of government rates disciplines banks' information choices. In the context of government interest rates, the key insight of the model is that banks choose their degree of opacity to minimize the transmission of monetary policy to their depositors. When safe outside option is attractive—i.e., when government rates are high—depositors become less tolerant of opacity, forcing banks to be transparent and offer higher returns to retain funding. When government rates are low, banks can afford to be fully opaque, as depositors have little incentive to withdraw. It is only at intermediate government rates that the bank optimally adopts partial opacity, balancing illiquidity and insolvency risk through portfolio allocation. This implies that changes in the broader interest rate environment can directly shape the nature of financial fragility—not just through asset valuations, but by altering banks' incentives to be transparent or opaque. In this way, the model provides a micro-foundation for why fragility, opacity, and maturity mismatch tend to co-move with interest rate cycles.

Related Literature. This paper contributes to a growing literature on the interplay between information disclosure, liquidity risk, and financial stability.[1]

It is well acknowledged that the banking system is opaque (see, e.g., Morgan (2002); Flannery et al. (2013)). The financial crisis of 2007–2008 emphasized the opacity of the financial system and prompted a line of research focused on the effects of financial transparency on systemic risk and the role of disclosure policies. Bouvard et al. (2015), Alvarez and Barlevy (2015), Goldstein and Leitner (2018), and Orlov et al. (2017) provide models that study the costs and benefits of disclosing bank-specific information. They show that increasing transparency is generally beneficial during financial crises but has ambiguous effects in normal economic times. Izumi (2021) highlights a related insurance–fragility trade-off arising from opacity in asset valuation, while Wei and Zhou (2021) show that anticipated regulatory disclosures can affect banks' funding conditions. These papers generally treat information disclosure as exogenous or policy-imposed, whereas we model it as a bank's endogenous choice.

Several studies highlight the role of information sensitivity in triggering bank runs and the endogenous dynamics of bank fragility. Goldstein and Pauzner (2005) and Morris and

[1]Our analysis builds on the tradition initiated by Diamond and Dybvig (1983), where banks provide liquidity insurance but are vulnerable to runs. While Diamond and Dybvig (1983) model runs driven by random liquidity shocks and coordination failures, we focus on information-driven fragility, where depositors withdraw based on noisy signals about bank asset performance. In addition, rather than treating opacity as a passive feature, we model banks as strategic designers of their information environment, balancing liquidity and solvency considerations.

Shin (2003) show that, with a fixed information structure, depositor signals critically affect the probability and nature of runs. He and Manela (2016) model rumor-based runs, showing how endogenous information acquisition by depositors can trigger liquidity crises even at solvent institutions. Dang et al. (2017) emphasize that opacity can preserve the liquidity of short-term debt by preventing information-sensitive withdrawals, while Dang et al. (2015) argue that financial stability often relies on maintaining investors' ignorance. Keister and Narasiman (2016) analyze how expectations and anticipated policy responses influence run dynamics, but focus on bailouts rather than the pricing of short-term debt. Our contribution is to focus on how banks strategically shape the information environment through opacity choices that balance interim liquidity risk and ex-post solvency.

Empirical evidence supports the importance of opacity in banking outcomes. Most directly related to our mechanism, Chen et al. (2022) documents that transparent US banks pay higher deposit rates. In particular, we find that in high interest rate environments, opaque banks pay a lower deposit rate compared to transparent ones. Chen et al. (2022) show that greater transparency increases the sensitivity of uninsured deposit flows to banks' performance signals, amplifying liquidity risk. This underscores the practical relevance of modeling opacity choices and their implications for financial stability.

Our work also relates to broader studies on information provision and disclosure incentives in financial markets. Frenkel et al. (2020) and Malenko and Malenko (2019) analyze how information supplied to market participants shapes voluntary disclosure and governance outcomes. While these papers focus on corporate settings, they similarly demonstrate how the design of the information environment shapes agent behavior and systemic dynamics.

Building on this literature, our main contribution is to model opacity as a strategic decision by banks. We link portfolio transparency choices to the trade-off between interim liquidity and ex-post solvency risks. We show that banks optimally choose intermediate levels of opacity, and that this choice varies systematically with market conditions, particularly with depositors' outside options. This approach offers a new perspective on the origins of financial fragility, emphasizing opacity as an equilibrium outcome of banks' optimization behavior.

The rest of the paper is organized as follows. In Section 2 we introduce the model set-up. In Section 3 we introduce the full information and no information benchmarks to highlight the role of opacity. Our main results are presented in Section 4. Section 5 discusses the implications of our model. Section 6 concludes.

2 The Model

The focus of our analysis is to understand how banks use opacity in their portfolios. To this end, we introduce a framework where a bank chooses her portfolio of risky loans not only to optimize her risk exposure but, crucially, to influence the information available to depositors. Naturally, information is particularly important for depositors when they have the bargaining power to use it. For this reason, we develop a model in which the return on deposits is set to maximize depositors expected payoffs, while the bank's bargaining power is reflected in her asset portfolio choice. This approach ensures that both the depositors and the bank make positive profits, leading to interesting trade-offs.

2.1 The Environment

Consider an economy with three dates, $t = 0, 1, 2$, and two types of agents. First, a bank that can finance long-term risky projects but has no funding. Second, households who own c units of endowment in aggregate but do not have direct access to risky projects. All agents are risk-neutral.

To invest in projects, the bank must raise funds from households in the form of *deposits*. In exchange for borrowing funds from the depositors, the bank issues a long-term deposit contract which matures at date 2 and pays face value D at maturity. Throughout the paper, we refer to D as the return on deposits.

Risky Projects There is a universe of long-term risky projects which the bank can invest in. All the project have a common scale c. Each project i has scale returns R_i at date 2. Project returns are independent draws from a uniform probability distribution $G(\cdot)$, with support $[0, 1]$.

Projects are of two types: *transparent* (T) and *unobservable* (U). The return, R_i, of a transparent project is perfectly revealed to depositors at date 1. In contrast, depositors receive no information about the return of an unobservable project. Let s_i denote the type of project i with $s_i \in \{R_i, \varnothing\}$, where $\varnothing$ represents that there is no information about the project.

Bank Portfolio Choice The bank raises funds from depositors and invests in projects at date 0. We assume that the bank can invest in at most two projects, $(1, 2)$, to keep the analysis tractable. The bank makes two decisions when choosing her portfolio.

First, she chooses a *portfolio allocation* $(\phi, 1-\phi)$, representing the fraction of the bank portfolio in projects 1 and 2, respectively, with $\phi \in [0, 1]$. Bank's portfolio allocation governs the return on the bank portfolio at date 2.

Second, she chooses a *portfolio composition*, $\mathbf{s} = (s_1, s_2)$, which denotes the project types in her portfolio. Bank's portfolio composition governs the interim information structure of depositors at date 1. If $s = (R1_1, R_2)$ we call the bank portfolio *transparent*, if $s = (R_1, \varnothing)$ we call it *opaque*, and if $s = (\varnothing, \varnothing)$ we call it *unobservable*.

Portfolio Return The return on bank portfolio at date 2 is given by

$$V(\phi) = \phi R_1 + (1-\phi) R_2.$$

As such, the cumulative distribution of return on bank's portfolio at date 2, $H(.; \phi)$, is determined solely by the portfolio allocation, ϕ, and is independent of bank's portfolio com-

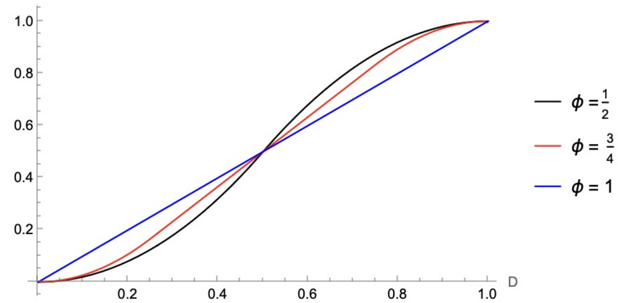

Figure 1: Cumulative Distribution of Bank's Portfolio Return $V(\phi)$

position.[2] Figure 1 depicts the cumulative distribution of return for three different values of portfolio allocation.

Note that the mean of the return distribution is $\bar{R} = \frac{1}{2}$, independent of the portfolio allocation, ϕ. However, the riskiness of the return distribution, as measured by second-order stochastic dominance, does depend on the portfolio allocation. The distribution of returns is symmetric around $\phi = \frac{1}{2}$: allocations at $\phi = 0$ or $\phi = 1$ correspond to *concentrated* or *fully undiversified* portfolios and yield the riskiest return distribution. In contrast, $\phi = \frac{1}{2}$ corresponds to a *perfectly diversified* portfolio, which minimizes return variance and generates the most concentrated distribution around the mean. Moving ϕ closer to $\frac{1}{2}$ makes the return distribution more concentrated and stochastically dominant in the second order. For this reason, we refer to portfolios with ϕ near $\frac{1}{2}$ as *well-diversified*.

Interim Information Structure of Depositors Depositors (might) receive information about the bank's portfolio return at date 1. In particular, the depositors observe a signal about any project in the bank portfolio that is transparent. For simplicity, assume that for each transparent project the interim information perfectly reveals the project return. This information is important as depositors face a withdrawal decision at date 1, which is governed by their information structure.[3]

Thus, bank's portfolio composition determines the informativeness of date 1 signal to

[2] In **??** we provide the details of the return distribution $H(z;\phi)$.

[3] Note that as the bank does not make any decision at date 1, bank's interim information structure is irrelevant.

bank depositors. When the portfolio is *transparent* (T), and $\mathbf{s} = (R_1, R_2)$, depositors receive perfect information about the bank's date-2 return already at date $t = 1$. When the portfolio is *unobservable* (U), and $\mathbf{s} = (\varnothing, \varnothing)$, depositors receive no information at date $t = 1$ about the bank's return at date 2. Finally, when the portfolio is opaque (O), depositors have partial information at date $t = 1$ about the bank's return at date 2. Without loss of generality, we let R_1 be the return of the transparent project, and R_2 the return of the unobservable project, so that $\mathbf{s} = (R_1, \varnothing)$.

Bank's portfolio allocation ϕ is also relevant in determining the extent of depositors' interim information iff bank's portfolio composition is opaque.

Early Withdrawal Decision of Depositors After observing information about returns according to the portfolio composition $\mathbf{s}$, depositors decide whether to demand their deposits early at date 1, or wait to receive D at date 2. We represent a depositor's decision at date 1 through a function

$$\omega(\mathbf{s}) = \begin{cases} 1 & \text{if depositor withdraws,} \\ 0 & \text{if depositor continues.} \end{cases}$$

In our setup all depositors are identical and there are no coordination concerns. We focus on symmetric equilibria where if one depositor finds it optimal to withdraw early, then all depositors withdraw early.

If depositors choose to withdraw early, they receive a redemption value $r < E(R_i)$. We assume $c < r$, which is sufficient to ensure that the participation constraint of depositors in lending to the bank at date 0 is satisfied.

We provide two different interpretations for r. First, if the bank faces early withdrawals, she has to sell her portfolio of projects to second-best users at a fire sale prices r, which is transferred to depositors entirely.[4] A second interpretation is that if depositors withdraw

[4]Projects are worth less to second-best users because of the misallocation mechanism proposed by Shleifer and Vishny (1992) and Kiyotaki and Moore (1997), and adopted by Lorenzoni (2008). As such, they can be sold only at a discount.

early, the bank is forced to liquidate her portfolio of projects early at one-to-one return, and the liquidation value c is transferred to depositors. At this point, the depositors have access to an outside short-term riskless investment opportunity that returns $\frac{r}{c}$ per unit investment at date 2. Under this interpretation, r represents the interest rate paid on government bonds.

What is important is that, in both interpretations, the bank lacks sufficient resources to meet early withdrawals while continuing its investment, and thus she fails early. Put differently, depositors' willingness to accept r as a redemption value renders the bank *illiquid* at date 1.

Payoffs If depositors choose to continue, they receive D at date 2 provided the bank's portfolio returns enough, that is, if $V(\phi) \geq D$. Otherwise, if $V(\phi) < D$, the bank is *insolvent* and enters costly bankruptcy at date 2, having insufficient resources to repay depositors when the projects mature. For tractability, we make the stark assumption that bankruptcy absorbs all the projects' payoff. In other words, the fraction that the depositors receive from the bank's portfolio is 0.[5]

Considering the decision depositors make at date 1, the payoff they receive at date 2 can be expressed as

$$(1-\omega(\mathbf{s})) \cdot \left(D \cdot \mathbf{1}_{\{V(\phi)\geq D\}}\right) + \omega(\mathbf{s}) \cdot r, \tag{1}$$

where $\mathbf{1}_{\{V(\phi)\geq D\}}$ is an indicator function such that it is equal to 1 if $V(\phi) \geq D$ and 0 otherwise.

The bank is the residual claimant on the return of her portfolio and receives at date 2

$$\max\{V(\phi) - D, 0\}. \tag{2}$$

Thus, the bank receives 0 both if she is illiquid and depositors withdraw at date 1 or if she

[5]The results are robust to the alternative assumption that the fraction that the depositors receive of the bank's portfolio is a small positive α in the event of bank default at date 2. The robustness results are available upon request.

is insolvent and default at date 2.

Timing At date 0, the bank chooses simultaneously the types of projects in her portfolio and the allocation for each of them, i.e. the opacity and diversification levels. Afterwards, at date 1, depositors may observe information about the projects, depending on the types of projects the bank chose. Subsequently, they decide whether to withdraw or continue with their investment with the bank. If depositors withdraw their money early at this point, they receive r and the bank gets 0. If depositors continue their investment, projects mature and returns are realized at date 2. If the bank is solvent at date 2, depositors are paid the return on deposits D and the bank gets $V(\phi)$. Otherwise, the depositors and the bank receive 0.

2.2 Equilibrium

Both bank and depositors are rational and internalize how their choices will affect others' actions and subsequent payoffs. Thus, the bank decides strategically the composition of project types, $\mathbf{s} = (s_1, s_2)$, in her portfolio and the portfolio allocation $(\phi, 1-\phi)$. In other words, the bank chooses $\mathbf{s}$ and ϕ to maximize her expected profit, as we describe in detail below.

Similarly, depositors decide strategically whether to withdraw early. For this decision to be meaningful and to interact with the bank's choice, we assume the bank offers depositors a competitive deposit contract. Specifically, the return on deposit, D, is set to maximize the depositors' expected surplus, given that they lend all their funds. This assumption ensures that both the bank and the depositors receive non-trivial payoffs.

In this set-up we use the following equilibrium concept.

Definition 1 (Equilibrium). *An equilibrium is a composition of project types* $\mathbf{s} = \{s_1, s_2\}$*, a portfolio allocation* $(\phi^*, 1-\phi^*)$*, a return on deposits* D^**, and depositors' continuation decision* $\omega^*(s_1, s_2)$ *such that*

1. *the continuation decision maximizes depositors' expected payoff at date* 1

$$\max_{\omega}\left\{(1-\omega(\mathbf{s}))\cdot D\cdot \Pr(D\leq V(\phi)\,|\mathbf{s})+\omega(\mathbf{s})\cdot r\right\};$$

2. *the return on deposits maximizes the depositors' expected payoff at date* 0

$$\max_{D}\mathbb{E}_{\mathbf{s}}\left\{(1-\omega(\mathbf{s}))\cdot D\cdot \Pr(D\leq V(\phi)\,|\mathbf{s})+\omega(\mathbf{s})\cdot r\right\};$$

3. *the project type decision and portfolio allocation maximize the bank's expected payoff at date* 0

$$\max_{\phi,\mathbf{s}}\mathbb{E}_{R_1,R_2}\{(1-\omega(\mathbf{s}))\cdot \max[(V(\phi)-D),0]\}.$$

Implicitly, the optimal withdrawal decision is a function of the return on deposits as well as the fraction of transparent and unobservable projects in the bank's portfolio, i.e. $\omega^*(s_1,s_2)=\omega^*(s_1,s_2;D,\phi)$. Similarly, the return on deposits is a function of the bank's projects type and portfolio allocation, i.e. $D^*=D^*(\phi,s_1,s_2)$. In the exposition below, we take these dependencies as implicit so not to burden excessively the notation.

2.3 Solution Concept

Characterizing the equilibrium involves two steps. The first step is to solve for the bank's portfolio allocation choice keeping portfolio composition $\mathbf{s}$ fixed. This implies finding ϕ^* for when the bank holds: i) only transparent project(s) with $\mathbf{s}=(R_1,R_2)$; ii) only unobservable project(s) with $\mathbf{s}=(\varnothing,\varnothing)$, iii) both an unobservable and a transparent project with $\mathbf{s}=(R_1,\varnothing)$.

In each of these three cases, we solve the model backwards. We characterize the depositors' optimal withdrawal decision, $\omega^*(\mathbf{s})$, and the optimal return on deposit, D^*, that satisfy conditions (1) and (2) in Definition 1, given a bank's portfolio allocation $(\phi,1-\phi)$. Then, we solve for the optimal portfolio allocation, $(\phi^*,1-\phi^*)$, taking into account that the bank understands that the depositors behave optimally.

In the second step, we characterize the bank's optimal choice of portfolio composition, taking into account the optimal allocation associated with each composition. This step determines the equilibrium degree of opacity and diversification when the bank jointly chooses how much information to reveal and how to allocate risk.

Solving the model delivers four variables that characterize equilibrium: bank's optimal portfolio composition $\mathbf{s}^*$ and optimal portfolio allocation $(\phi^*, (1-\phi^*))$; the optimal return on deposit, D^*; and the depositors early withdrawal decision ω^*. While the first three variables are *ex-ante* equilibrium outcomes, the early withdrawal decision represents an *interim* outcome.

In addition, it is useful to include in the equilibrium analysis the probability of early withdrawal or *probability of illiquidity* of the bank at date 0, defined as

$$\begin{aligned}\Omega(\phi, D) &= \Pr(\text{bank fails at date } t = 1) \\ &= \Pr(\omega\,(\mathbf{s}) = 1), \end{aligned} \tag{3}$$

as well as the *probability of insolvency* of the bank at date 1, defined as

$$\begin{aligned}\Psi(\phi, D; \mathbf{s}) &= \Pr(\text{bank fails at date } t = 2 | \text{reaching date } t = 2) \\ &= \Pr(V(\phi) < D | \omega\,(\mathbf{s}) = 0). \end{aligned} \tag{4}$$

From a high-level perspective, the bank wants to maximize its profit. High profitability has three components: (1) staying liquid in the interim date 1, i.e. low Ω; (2) staying solvent ex-post at date 2, i.e. low Ψ; (3) paying a low rate of return on deposits at date 2, i.e. low D. Crucially, we distinguish between two forms of fragility—illiquidity and insolvency—and, perhaps counterintuitively, minimizing one does not necessarily help minimize the other. As we show, these objectives can come into conflict, and then the bank faces a trade-off in her portfolio choice.

To clarify the bank's incentives, we begin by characterizing the optimal portfolio allocation taking the information structure as given. We study transparent and unobservable portfolios as benchmark cases. We then turn to the bank's joint choice of portfolio composition and allocation, which determines the equilibrium degree of opacity.

3 Benchmark Cases

As a preamble to the full equilibrium characterization, we consider two special cases: a (fully) transparent portfolio and a (fully) unobservable portfolio. In both settings, the bank chooses a portfolio allocation $\phi \in [0, 1]$ across two risky projects and offers depositors a contract characterized by a repayment D at date 2. The bank's decision only affects the distribution of her portfolio returns, and not depositors' information structure. These benchmark cases clarify how the bank's incentives interact with the information structure, particularly how they depend on whether depositors can observe the return realization at the interim date.

Throughout the analysis of these benchmark cases, we adopt the convention that the bank's optimal portfolio allocation ϕ lies in the interval $[0, \frac{1}{2}]$, exploiting the symmetry of the return distribution. Accordingly, the boundary cases $\phi = 0$ and $\phi = 1$ are treated as equivalent and represent fully undiversified portfolios. All proofs are provided in the Appendix.

3.1 Transparent Portfolio

We start by analyzing the bank's portfolio allocation choice when both projects are transparent, with $\mathbf{s} = (R_1, R_2)$. Depositors have perfect information about the return on bank's portfolio when making their decision at date 1. at date 1. Thus, they continue to fund the bank if they are certain that the bank does not default at date 2. Conversely, they withdraw their deposit early when they learn that the bank's portfolio returns will be insufficient to cover their return on deposit. In other words, the depositors' withdrawal decision at date 1 is as follows

$$\omega(\mathbf{s}) = \begin{cases} 1 & \text{if } V(\phi) < D \\ 0 & \text{if } V(\phi) \geq D \end{cases}.$$

The bank understands that if depositors choose not to withdraw at date 1, it is certain that she will remain solvent at date 2 and will therefore receive the residual payoff $(V(\phi)-D)$. In other words, the probability of insolvency conditional on depositors not withdrawing, is $\Psi = 0$. This implies that the date-0 probability that the bank is illiquid and that depositors withdraw early is $\Omega = \Pr(V(\phi) < D)$.

Since insolvency is no longer a concern once the bank is continued, the only risk she is facing is illiquidity. Thus, for a given return on deposits D, the bank's expected payoff at date 0 is given by

$$\mathbb{E}_{R_1,R_2}\left[(V(\phi) - D)\middle|V(\phi) \geq D\right] \cdot \Pr\left[V(\phi) \geq D\right]. \tag{5}$$

For a fixed return on deposit, the bank benefits from a lower probability of early withdrawals, which translates one-for-one into a lower risk of illiquidity. At the same time, as the residual claimant, she captures the excess return on the portfolio above the promised payment to depositors. The following lemma formalizes this trade-off.

Lemma 1. *For any fixed portfolio allocation ϕ, the bank's expected payoff is decreasing in the return on deposits, D.*

Lemma 1 simply states that conditional on the return distribution of the bank's portfolio, bank expected payoff in Equation (5) is decreasing in the return on deposits. This is intuitive as the return on deposits impacts bank's expected profit through two channels that go in the same direction. First, since a transparent portfolio implies perfect information on portfolio returns, depositors' withdrawal decision is perfectly aligned with the bank's (in)solvency risk. They will withdraw early if and only if the (to-be-realized) bank's future return is below the return on deposits. As such, a higher return on deposits leads to a higher probability of early withdrawal and thus lower bank expected profit. Second, a higher return on deposits implies that whenever the depositors do not withdraw early, they receive a higher share of the portfolio return at date 2, which in turn means that bank's profit will be lower.

Since the return on deposits D is not exogenous, it must solve the following maximization problem

$$\max_{D} \quad D \cdot \Pr\left(V(\phi) \geq D\right) + r \cdot \Pr\left(V(\phi) < D\right). \tag{6}$$

The optimal return on deposits depends on both the depositors' outside option r and the bank's portfolio allocation ϕ. A higher outside option raises the return depositors require, while well-diversification - portfolio allocations closer to $\frac{1}{2}$ - makes repayment more likely and allows the bank to offer a lower return on deposits.

To complete the equilibrium characterization, we characterize the bank's optimal portfolio allocation. The following proposition shows how the equilibrium allocation depends on the depositors' outside option r, accounting for the endogenous determination of deposit rates and withdrawal decisions.

Proposition 1 (Equilibrium Transparent Portfolio). *When bank's portfolio composition is transparent, there exists a threshold $\bar{r}_T$ such that*

1. *If $r < \bar{r}_T$, the bank chooses a well-diversified portfolio allocation that minimizes the return on deposits at date 2.*

2. *If $r \geq \bar{r}_T$, the bank chooses a fully undiversified portfolio allocation.*

When the portfolio is transparent, depositors observe interim returns perfectly. As a result, they withdraw if and only if the bank would otherwise be insolvent at maturity. Conditional on continuation, insolvency never occurs, so illiquidity and insolvency coincide: whenever the bank remains liquid at date 1, it is solvent at date 2.

Because there is no information friction at the withdrawal stage, the bank effectively faces a single-dimensional problem. Portfolio choices affect both the single-failure probability and the return promised to depositors, but there is no trade-off between minimizing illiquidity and minimizing insolvency. The bank, therefore, chooses its portfolio allocation solely to balance failure risk and funding costs.

To build intuition about the bank's portfolio allocation, it is useful to consider a simple one-period debt contract. Once returns are realized, the bank is either solvent or insolvent, depending on whether the portfolio return R exceeds the rate of return on debt D. If the bank is solvent, she receives the residual payoff $R - D$.

When the return on debt D is endogenous, minimizing the probability of insolvency also minimizes what the bank must repay to depositors. A lower probability of default makes repayment more likely, so depositors are willing to accept a lower promised return. This mechanism underlies the general intuition for bank's incentive to diversify. A higher diversification, $\phi \approx \frac{1}{2}$, makes the return distribution second-order stochastically dominant, and a reasonably high return is more certain. In other words, diversification makes repayment more predictable and lowers funding costs.

In contrast, when the return on deposits is exogenously fixed, the bank's incentives change: she only cares about upside outcomes. In this case, diversification compresses the tails of the return distribution, reducing the likelihood of extreme positive outcomes. Because the promised return does not adjust, these high-return states are less likely, despite being the ones that are most profitable for the bank. As a result, when D is high, the bank avoids diversification and instead concentrates its portfolio, engaging in risk-shifting behavior to maximize the chance of extreme positive outcomes.

Although in our model the return on deposits is determined endogenously, the same risk-shifting behavior applies when the depositors' outside option r is high. That is so because r acts as an exogenous lower bound on the required D. Thus, when r is high, the only feasible rates of return on debt are high as well. Put differently, when r is sufficiently large, the bank optimally shifts risk by holding a more concentrated portfolio.

This argument highlights two points. First, risk-shifting behavior is independent of the demandable nature of the deposit contract: when the required return is sufficiently high, the bank optimally concentrates its portfolio, just as in a standard one-period debt setting. Second, when the portfolio composition is transparent, and r is not too high, the bank's incentives to minimize the probability of illiquidity, the probability of insolvency, and the return on deposits D are fully aligned; only when r becomes sufficiently large do these objectives come into conflict, and risk-shifting incentives dominate.

3.2 Unobservable Portfolio

We now analyze the bank's portfolio allocation when both projects are unobservable, with $s = (\varnothing, \varnothing)$, and so depositors receive no interim information about portfolio returns. As a result, their withdrawal decision at date 1 is fully determined at date 0.

Specifically, for any portfolio allocation and return on deposits D, both determined at date 0, depositors will keep their deposits in the bank at date 1 if and only if the amount of the expected repayment at date 2, $D \cdot Pr(V(\phi)D)$, exceeds the reservation value, r. Otherwise, they withdraw their deposit and receive r.

Thus, the depositors' withdrawal decision at date 1 is known at date 0, when the bank chooses her portfolio allocation ϕ, and it is given by

$$\omega(\mathbf{s}) = \begin{cases} 1 & \text{if } D \cdot \Pr\left(V(\phi) \geq D\right) < r \\ 0 & \text{if } D \cdot \Pr\left(V(\phi) \geq D\right) \geq r \end{cases} .$$

This implies that at date 0 that bank faces two possible outcomes. If the bank portfolio allocation ϕ and the pursuant return on deposits D are such that

$$D(\phi) \cdot \Pr\left(V(\phi) \geq D(\phi)\right) \geq r, \tag{7}$$

then the depositors do not withdraw early, and the probability of illiquidity is 0. The only risk remaining for the bank is the risk of insolvency, $(1 - \Pr(V(\phi) \geq D))$. In this case, the bank's expected payoff at date 0 is given by

$$\mathbb{E}_{R_1,R_2}\left[V(\phi) - D)|V(\phi) \geq D\right] \cdot \Pr\left[V(\phi) \geq D\right]. \tag{8}$$

Inequality (7) is essentially a participation constraint of depositors. Thus, if violated, the bank gets 0 at date 2.

Note that for any fixed return on deposits D, the bank's expected payoff in Equation (8) is identical to that in the transparent portfolio case, as given in Equation (5). This equivalence arises because the bank faces only one source of risk in both settings. Consequently, Lemma 1 applies equality with an unobservable portfolio, provided the participation constraint (7) is satisfied.

With an unobservable portfolio, unlike the transparent benchmark, there may not exist a return on deposits that persuades depositors to continue funding the bank, regardless of the portfolio allocation the bank chooses. In particular, the return on deposits must solve the following problem

$$\max_D D \cdot \Pr\left(V(\phi) \geq D\right). \tag{9}$$

At date 0, if the solution to problem (9) satisfies the participation constraint (7), then it is certain that depositors will continue with the bank. Otherwise, they will choose to withdraw. Without loss of generality, we assume that if the depositors are anticipated to withdraw with probability one, they do not lend to the bank in the first place.

When the depositors' outside option r is sufficiently attractive, it is straightforward to verify that Problem (9) admits no feasible solution for any portfolio allocation ϕ chosen by the bank. In this case, no return on deposits can satisfy depositors' participation constraint, and depositors optimally withdraw regardless of how the bank allocates risk.

As a consequence, in the unobservable benchmark, depositors effectively commit at date

0 to their withdrawal decision. Because no information is revealed at the interim date, depositors lack the ability to condition their decision on realized portfolio returns. The return on deposits therefore compensates depositors for this lack of flexibility, which in some cases requires a promised repayment that exceeds what the bank can profitably offer.

For any return on deposits that satisfies Problem (9) and depositors continue with the bank, the following proposition characterizes the bank's portfolio allocation choice.

Proposition 2 (**Equilibrium Unobservable Portfolio).** *When bank's portfolio composition is unobservable, there exists a threshold* $\bar{r}_U$ *such that*

1. *If* $r < \bar{r}_U$*, the bank chooses a well-diversified portfolio allocation that minimizes the return on deposits at date* 2*.*

2. *If* $r \geq \bar{r}_U$*, the bank cannot implement any portfolio allocation that prevents depositors from withdrawing early.*

When the depositors' outside option r is sufficiently low, the bank can choose a return on deposits that satisfies the participation constraint in Problem (9). In this case, depositors optimally continue at the interim date, and the probability of illiquidity is zero. The bank then faces only insolvency risk at maturity. As in the transparent benchmark, the bank's objective in setting the portfolio allocation ϕ is to minimize a single-dimensional risk of failure and the return she must promise depositors. The results in the bank holding a well-diversified portfolio that lowers both the probability of insolvency and the rate of return of deposits.

As the outside option r increases, satisfying the participation constraint becomes more difficult. The bank must offer a higher expected repayment to induce depositors to continue, which raises the required return on deposits and reduces profitability. When r is sufficiently high, Problem (9) admits no feasible solution for any portfolio allocation. In this region, no return on deposits can compensate depositors for committing to an unobservable portfolio.

As a result, an unobservable portfolio becomes infeasible: depositors withdraw with certainty at the interim date, and the bank fails regardless of how it allocates risk.

Taken together, the transparent and unobservable benchmarks highlight a common limitation: when portfolio composition is fixed, the bank cannot fine-tune the information available to depositors through its choice of portfolio allocation. Depositors observe either full interim information under transparency or no information under unobservability, independently of how the bank allocates its portfolio.

In both benchmarks, portfolio allocation affects only the distribution of future returns, which in turn determines solvency, expected profits, and funding costs. As a result, the bank's allocation choice typically reflects standard debt-contract incentives, leading it to favor diversification or, when required returns are sufficiently high, to engage in risk shifting. Crucially, this behavior arises without allowing the bank to shape depositors' interim beliefs.

We next turn to the general case in which the bank jointly chooses portfolio composition and allocation, allowing opacity to emerge endogenously. With an opaque portfolio, portfolio allocation affects not only the distribution of returns and solvency but also the informativeness of depositors' interim signals. As a result, the bank faces a genuine trade-off between liquidity and solvency, accepting a higher risk of early withdrawal in exchange for cheaper funding.

4 Main Result: Emergence of Opaque Banks

In this section, we characterize the bank's optimal joint choice of portfolio composition and allocation. Put differently, we determine the degree of portfolio opacity that emerges endogenously in equilibrium. Proposition 3, the main result of the paper, formalizes this outcome.

Proposition 3 (Jointly Optimal Portfolio Composition and Allocation). *When the bank chooses both the portfolio composition and the portfolio allocation simultaneously, there* *exist values* $\bar{r}_U < \bar{r}_O$ *such that:*

1. *when* $r \leq \bar{r}_U$*, the bank holds a well-diversified unobservable portfolio,*

2. *when* $\bar{r}_U < r \leq \bar{r}_O$*, the bank holds an opaque portfolio with imperfect diversification,*

3. *when* $r > \bar{r}_O$*, the bank holds a fully concentrated transparent portfolio.*

Proposition 3 nests two well-established insights. The first is that keeping depositors uninformed can prevent banking panics (Dang et al., 2017). To see how this arises in our setting, consider Figure 2, which depicts the three regimes that emerge in equilibrium as a function of the redemption value of deposits, r. In region i, the leftmost regime, the bank optimally holds an unobservable and well-diversified portfolio. By choosing an unobservable composition, she withholds all interim information from depositors; by selecting a well-diversified allocation—consistent with Proposition 2—she ensures repayment is sufficiently likely to persuade depositors to finance the bank and avoid withdrawing early. Proposition 3 clarifies, however, that this mechanism is at work only when the redemption value r is low—i.e., when depositors' outside option is relatively unattractive.

The second insight is illustrated in region iii, the rightmost regime. Here, the bank chooses a transparent portfolio and engages in risk shifting, foregoing diversification entirely. When depositors have a very high outside option, the only way to convince them to keep their deposits in the bank is to assure them that, if they wait until date 2, they will receive a high rate of return. The bank achieves this by holding a transparent portfolio. Even so, the high outside option implies that depositors still withdraw frequently and demand a high promised return. Consistent with Proposition 1, in response to this high required deposit rate the bank optimally engages in risk shifting and holds a fully concentrated portfolio in region iii.

The main takeaway from Proposition 3, however, is that in equilibrium, the bank chooses an opaque portfolio as a lever to substitute between illiquidity and insolvency risk. Indeed, region ii of Figure 2 shows that the bank chooses an opaque portfolio for a wide range of

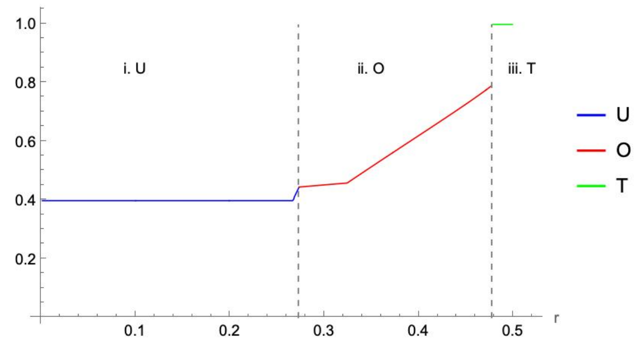

Figure 2: Bank's Portfolio Holding in Equilibrium

The three regions correspond to the bank's portfolio composition in equilibrium. In region i the bank holds an unobservable portfolio (U); in region ii an opaque portfolio (O); and in region iii a transparent portfolio (T). In the case of O, ϕ^* represents the fraction invested in the transparent project. The line is color-coded based on the regions to facilitate visual interpretation.

intermediate values of r, the outside option of depositors. In this region, r is high enough that, without interim information, depositors would panic and withdraw early with certainty, making an unobservable portfolio infeasible. Yet, r is not sufficiently high as to justify holding a (fully) transparent portfolio: with full information, depositors would require an excessively high rate of return. As a result, neither extreme — full information nor no information - is optimal. The bank's best response is to hold an opaque portfolio, which is partially informative.

Importantly, Proposition 3 has a counter-intuitive implication: unlike in the transparent or unobservable cases, an opaque portfolio makes it impossible for the bank to simultaneously minimize both illiquidity and insolvency risks. Choosing opacity, therefore, requires the bank to tolerate a high likelihood of early failure due to illiquidity. Yet, as the proposition shows, she finds this exposure optimal as it allows her to reduce insolvency risk and lower the long-term cost of funding.

This observation is made evident if we look at the rate of deposit and withdrawal probability in equilibrium, as depicted in Figure 3. Specifically, at the boundary between regions ii and iii. Since r represents what the depositors receive if they withdraw early, one would expect that an increase in r raises the early withdrawal probability — and thus the probability of illiquidity. Figure 3 confirms this intuition for nearly all values of r, except in the

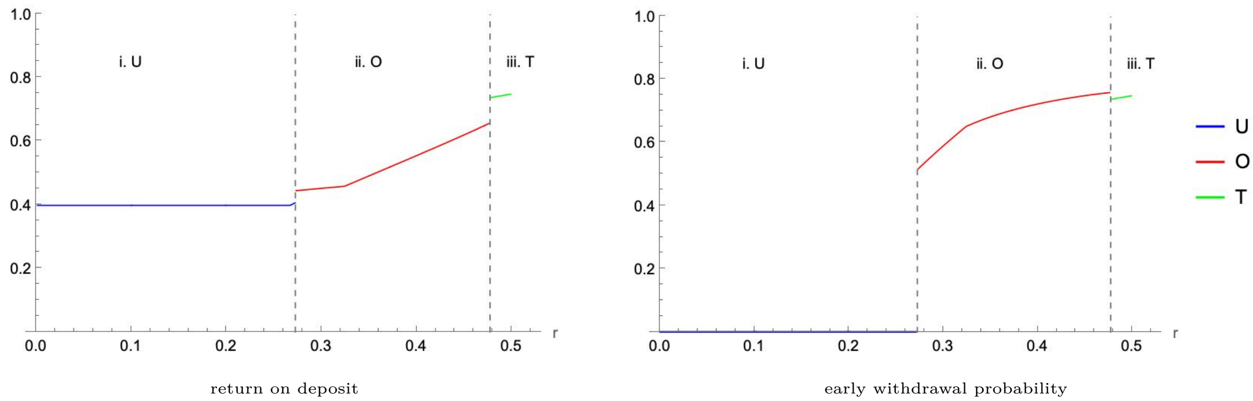

Figure 3: Equilibrium outcomes, return on deposits and depositors' early withdrawal probability

transition from region *ii* to *iii* - precisely where the bank switches its portfolio composition from opaque to transparent.

Going from opacity to transparency is accompanied by a decrease in the withdrawal probability and requires high levels of r. To understand why this happens, note that in region *ii*, on the left of the boundary, the bank chooses an opaque portfolio, and the lack of information to depositors leads to frequent early failures. However, the bank pays a relatively low return on deposits when solvent. As r increases past the threshold, withholding information becomes too costly, and the bank switches to a transparent portfolio. At this point, the probability of illiquidity actually declines, despite the increase in r. Transparency enables depositors to demand a sufficiently high return at date 2 (see the left panel in Figure 3), which in turn makes them willing to keep their funds in the bank, thereby reducing the risk of early withdrawal.

With an opaque portfolio, an alternative interpretation of the bank's optimal choice of portfolio allocation ϕ is as a *compound lottery.* The *outer lottery* corresponds to bank illiquidity—whether the bank fails early at date 1 or not. The *inner lottery* corresponds to bank solvency—whether the bank stays solvent at date 2 and if so, how much she pays the depositors. Ordinarily, one would assume that less frequent interim failures—a less risky outer lottery, is coupled with less frequent ex-post failures and a lower long-term return on deposits—a less risky inner lottery.

The model shows that this coupling breaks down when the depositors can realize a high value for withdrawing their deposits early from the bank. In this case, to prevent early withdrawal and stay liquid—a *less* risky outer lottery, the bank has to commit to a very high long-term return on deposits—a *more* risky inner lottery. However, the bank has a strong preference for a less risky inner lottery with low rate of return on deposits, as that is the only time that she is paid.

This preference leads to the novel insight of the model: an *opaque* portfolio allows the bank to shift risk from insolvency to illiquidity. That is, an opaque portfolio in which the allocation ϕ maximizes bank's profit enables her to engineer a riskier outer lottery (illiquidity) and a safer inner one (insolvency), accompanied by a lower long-term return on debt. The optimal portfolio allocation leads to a low promised return and a high probability of solvency conditional on remaining liquid, but at the cost of frequent early failures. In this sense, the bank chooses to expose herself to *excessive fragility*.

Finally, Figure 2 shows that in equilibrium, transparency of bank portfolio increases when r is higher. In particular, in region (ii), the region where bank portfolios are opaque, ϕ^* is monotonically increasing from left to right, and an increase in ϕ^* corresponds to a higher (lower) degree of transparency (opacity). Comparing this to the right panel of Figure 3 illustrates that equilibrium portfolio transparency increases the risk of illiquidity, consistent with Chen et al. (2022).

5 Bank Fragility and Government Interest Rate

A large body of research has examined the sources of bank fragility and their consequences for the financial sector. Since the global financial crisis of 2008, swift shifts in interest rates—from prolonged near-zero levels to the steep increases seen in the post-pandemic period—have emerged as key stressors on banks' funding costs and run risk.

In this section, we analyze how bank opacity interacts with different interest-rate environments to influence the nature and extent of bank fragility. In our framework, the government rate determines depositors' outside option and therefore shapes the return that banks must promise to attract funding. Because the bank can use opacity to influence how informative interim signals are, changes in the government rate affect not only funding costs but also the bank's optimal choice of transparency.

As a result, interest-rate fluctuations alter the trade-off between liquidity and solvency in a way that does not arise under either fully transparent or fully unobservable portfolios. The following example illustrates the mechanism in its simplest form. It highlights the novel tension faced by a bank holding an opaque portfolio: strengthening solvency incentives by reducing insolvency risk can come at the cost of increasing the probability of early withdrawals, and vice versa. This liquidity–solvency trade-off is the defining feature of opacity and does not arise under the two benchmark compositions.

Example 1 (**Bank Portfolio Allocation Trade-off with an Opaque Portfolio**). *If the portfolio composition is opaque, there is a range $r \in [r_l, r_h]$ for which the optimal portfolio allocation ϕ^* does not minimize neither the probability of illiquidty nor the return on deposits.*

This example demonstrates the trade-off described in Section 4 that is specific to opaque portfolios: A bank with an opaque portfolio does not choose a portfolio allocation that minimizes the return on deposits, unlike in the transparent or unobservable portfolio cases. While she benefits when the return on deposits is low—since she is the residual claimant on the portfolio—she understands that this may increase the likelihood of early withdrawals. Conversely, reducing illiquidity risk requires a return on deposits that is too high from her perspective, thereby lowering her expected payoff. As the example shows, she avoids both extremes, choosing instead a portfolio allocation that balances the trade-off rather than minimizing either risk in isolation.

This example highlights the misalignment between two core determinants of the bank's payoff and spells out the central trade-off that emerges between the probability of illiquidity Ω on one side, and the probability of insolvency Ψ and the corresponding bank residual profit on the other side.

To see this, recall that the bank's profit depends on three components: the likelihood of early liquidation, Ω, as defined in Equation (3), the probability of default at maturity, Ψ, as defined in Equation (4), and the promised return on deposits, D. The bank's ex-ante expected payoff can be written as:

$$\big(1-\Omega(\phi,D)\big)\times\mathbb{E}_{R_1}\left\{\big(1-\Psi(\phi,D;R_1)\big)\times\mathbb{E}_{R_2}\Big[(V(\phi)-D)|V(\phi)>D\Big]\right\}.$$

In contrast to the transparent and unobservable cases—where the bank faces either illiquidity or insolvency risk, but not both—an opaque portfolio exposes her to both forms of fragility simultaneously. Crucially, decreasing Ω and Ψ concurrently can be conflicting. This is because the portfolio allocation ϕ simultaneously determines (i) the return distribution $V(\phi)\sim\phi G(\cdot)+(1-\phi)G(\cdot)$ and (ii) the information available to depositors. When these two components affect the bank's payoff in opposing ways, a trade-off arises. Opacity then becomes a strategic lever the bank uses to substitute between insolvency and illiquidity risk in a way that maximizes expected profits.

We now explain these two impacts separately. First, like the unobservable and transparent portfolio composition, the portfolio allocation ϕ determines the *return distribution* of bank portfolio at date 2, which in turn determines bank solvency, Ψ, and the long-term rate of return required by the depositors D, through that. Second, unlike the unobservable and transparent portfolio compositions, the portfolio allocation also governs the *depositor information* with an opaque portfolio. How much information depositors have determines their early withdrawal decision, which in turn determines illiquidity risk, Ω.

It follows that when the portfolio composition is opaque, the choice of portfolio allocation involves a trade-off. On the one hand, there is a diversification channel which is independent of the portfolio composition, as it is present in the unobservable and transparent portfolios as well. As $\phi\to\frac{1}{2}$ the ex-post return distribution becomes more diversified, which implies

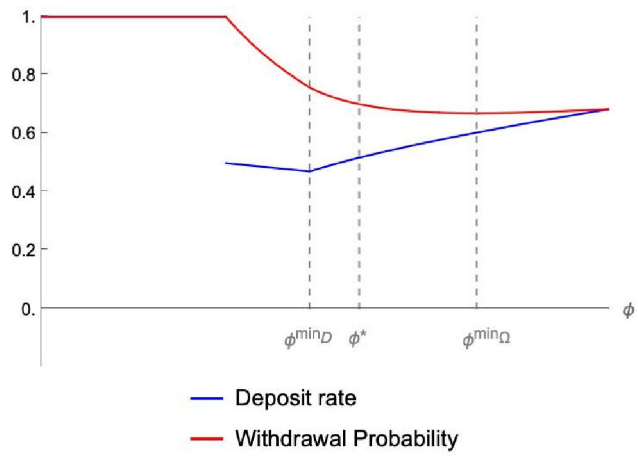

Figure 4: Equilibrium outcomes for an opaque portfolio when $r = 0.37$. This figure depicts the return on deposits and the withdrawal probability as a function of different choices of portfolio allocation ϕ. The optimal portfolio allocation chosen by the bank ϕ^*, as well as the portfolio allocations that minimize the return on deposits and default probability, are indicated on the x-axis - $\phi^{\min_D}$ and $\phi^{\min_\omega}$, respectively.

a lower D, and thus a lower Ψ, on average. However, a lower ϕ reduces the precision of depositors' information and makes them withdraw more to avoid the risk of bank becoming insolvent and not being paid, increasing the Ω. Thus, there is a misalignment between Ω and Ψ if $\phi \to \frac{1}{2}$ to increase the chance of bank solvency.

On the other hand, a high ϕ improves depositor information and allows them to withdraw more effectively, when they are less likely to be paid if the bank continues to operate until date 2. This in turn reduces the probability of early withdrawal and lowers Ω. On the other hand, it makes the ex-post return less diversified, increasing the required long-term return on debt D and the probability of bank insolvency Ψ, on average.

Put differently, when $\phi \to \frac{1}{2}$, despite the favorable low probability of insolvency and low return on deposit, depositors' early withdrawal decision is done with mediocre information. It is quite imprecise and destroys sizeable ex-post value. The imprecision of early withdrawal decision necessitates a high Ω, something the bank dislikes. Alternatively, when ϕ is large, depositors are better informed about likelihood of ex-post default given the predetermined D. As such, they withdraw their deposits efficiently, exactly when $\Psi(R_1)$ is high–or equivalently $(1-\Psi(R_1))$ is low. Effective withdrawal enables them to withdraw less frequently, decreasing

Ω, something the bank likes.

Example 1 shows that in equilibrium, the bank resolves this trade-off in a way that exposes it to defaulting early too frequently. In other words, bank chooses a "degree of opacity," ϕ, which does not minimize the probability of early withdrawal, i.e. bank illiquidity. One can think of this as a novel form of risk-shifting behavior: the bank loads on short-run risk of failure to decrease its long-run risk of failure.

Recall that r, i.e. the short-term return on deposits, represents the outside option of depositors if they decide to withdraw their deposits before maturity. One possible interpretation is that depositors can withdraw their deposits from the bank at date 1 (at par) and invest in the government interest rate, r_f. The bank would need to liquidate the project early one-for-one to pay back each unit of deposits. With this interpretation, r will be an affine transformation of the government interest rate.[6] To avoid introducing new notation, we refer to r as the government interest rate in this section.[7]

In order to relate our results to the existing empirical evidence and distinguish the novel findings of the model, it is insightful to compare the equilibrium outcome of the model vis-a-vis a *transparent portfolio with optimal allocation*, at different levels of interest rates. Figure 5 illustrated this comparison. [8]

The most important observation is that independent of the prevailing monetary policy rate, a transparent bank uniformly pays a higher deposit rate compared to an opaque bank, consistent with findings of Chen et al. (2022). However, the impact of opacity on bank fragility is non-uniform. In low interest rate environments, opacity makes bank portfolio information-insensitive and decreases the probability of illiquidity. However, this pattern reverses in high interest rate environments: opaque banks face more early withdrawal than

[6]Specifically, $r = c(1 + r_f)$, where c is the exogenous scale parameter.

[7]Furthermore, we normalize the interest rate on government bonds at date 0 to zero. As we assume that agents do not discount the future, this assumption is without loss of generality. Note that in this framework, discounting is equivalent to reducing bank payoff R at date 2 compared to r. Given that our results are for a general value of R, the no discounting assumption is without loss of generality as well.

[8]Note that an *unobservable portfolio with optimal allocation* only exists for low r. It coincides with the equilibrium outcome in region (i) of Figure 5.

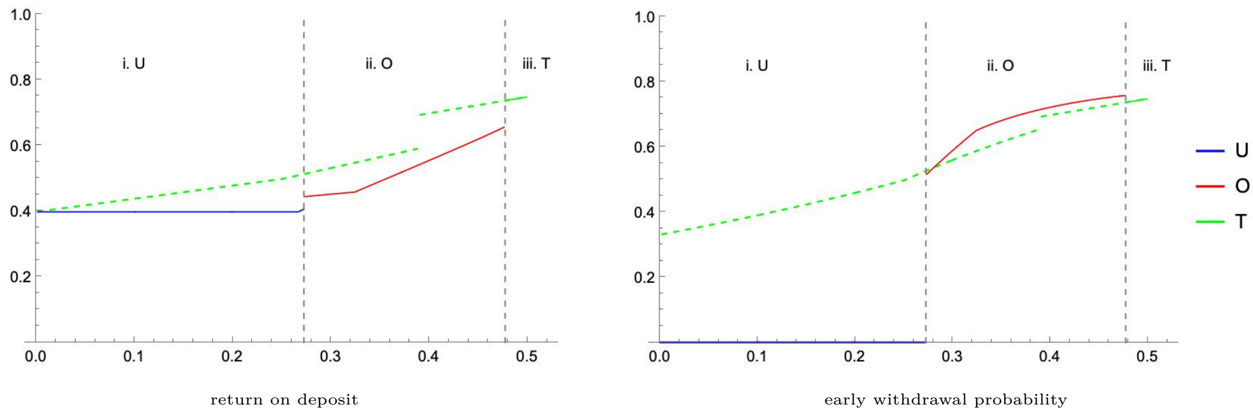

Figure 5: Equilibrium return on deposits and probability of early withdrawal: Comparison with the optimal transparent portfolio

transparent banks if government interest rates are sufficiently elevated.

The left half of both panels, region i in blue, corresponds to the mechanism proposed by Dang et al. (2017)—opacity preserves bank liquidity by preventing information sensitive early withdrawal. Our model generalizes this intuition when banks are not only subject to illiquidity but also to insolvency; that is, they face distinct possibilities of interim and ex-post failure. We show that by withholding all information from depositors, banks are able to improve chances of staying both liquid and solvent. When interest rates are low, keeping depositors uninformed prevents information-driven panic runs and allows a reasonably low long-term return on deposits, concurrently. Figure 5 exhibits both of these outcomes. The left panel shows that information-driven early withdrawals are less frequent with an unobservable portfolio compared to a transparent portfolio. The right panel illustrates that the bank is further able to attract cheaper deposits with an unobservable portfolio.

The critical observation is that this argument is relevant only in *low interest rate environments.* In fact when interest rates are *moderately high*, in region ii, opacity leads to excessive fragility of the financial sector. The right panel of Figure 5 manifests that for intermediate levels of interest rates, the bank optimally chooses an opaque portfolio—the red portion of the plot. In this region, the probability of early withdrawals is higher than the probability of early withdrawal in a (hypothetical) transparent portfolio with optimally chosen portfolio allocation. The reason why the bank chooses to expose herself to excessive early failure is that in return, she can offer a lower long-term return to depositors if she stays liquid, as evident in region *ii* of the left panel of Figure 5.

In summary, when interest rates are bounded away from zero, an *intermediate* degree of opacity allows banks to earn more profits by finetuning the amount of information that they convey to depositors. They provide depositors with sufficiently low information that they do not require excessively high rate of return, but sufficiently high that they have enough confidence in the bank to lend to her. In other words, bank opacity does not guarantee stability of the financial sector. In fact, with moderate interest rates, banks use opacity as a tool to shift risk forward: They are willing to suffer excessive information-driven early withdrawals in exchange for offering a low long-term deposit rate and staying solvent conditional on weathering the deposit withdrawals in the short run.

6 Conclusion

This paper develops a theory of opacity as a deliberate choice in banking. In our model, the bank selects not only which assets to hold, but also how much information to reveal to depositors through its portfolio composition and allocation. When the portfolio is opaque—mixing transparent and unobservable assets—allocation affects both the return distribution and depositor beliefs, creating a trade-off between illiquidity and insolvency.

We show that the bank exploits this trade-off strategically. Under certain funding conditions - modeled as different interest rate environments, she chooses an opaque, underdiversified portfolio that raises the likelihood of early liquidation. This fragility is intentional: by increasing the risk of runs, the bank reduces deposit rates and improves solvency. Opacity becomes a tool to substitute short-term failure for long-term risk, maximizing her expected surplus at the depositors' expense.

By endogenizing the information structure, the model reveals how fragility can be optimally engineered, not merely avoided. This perspective opens new avenues for studying how institutions use opacity, signal precision, and information control to shape their funding environment—particularly in the presence of systemic risk, regulation, or competition.

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Appendix

A Proofs

Characterization of the Portfolio Return Distribution

The proofs of the results in the main text use the cumulative distribution of return on the bank's portfolio as date 2. We next characterize it.

Recall bank's portfolio return $V(\phi) = \phi R_1 + (1-\phi)R_2$, where R_1 and R_2 are independent random variables uniformly distributed in $[0, 1]$, and $\phi \in [0, 1]$.

$V(\phi)$ has the cumulative distribution function $H(z; \phi)$,

$$H(z;\phi) = \begin{cases} \frac{z^2}{2\phi(1-\phi)} & z < \phi \\ (1-\phi)^{\{-1\}} \left(z - \frac{\phi}{2}\right) & \phi \leq z \leq 1-\phi \\ 1 - \frac{(1-z)^2}{2\phi(1-\phi)} & z > 1-\phi \end{cases} \tag{A.1}$$

and the probability density function,

$$h(z;\phi) = \begin{cases} \frac{z}{(1-\phi)\phi} & z < \phi \\ \frac{1}{1-\phi} & \phi \leq z \leq 1-\phi \\ \frac{1-z}{(1-\phi)\phi} & z > 1-\phi \end{cases} \tag{A.2}$$

Note that $H(z; \phi)$ distribution is symmetric around $\phi = \frac{1}{2}$. Thus, we consider the case $0 \leq \phi \leq \frac{1}{2}$ in the proofs. The case $\frac{1}{2} \leq \phi \leq 1$ follows by symmetry.

Proof of Lemma 1

Consider bank's expected payoff when she holds a transparent portfolio,

$$
\begin{aligned}
&\mathbb{E}\left[(V(\phi) - D) | V(\phi) \geq D\right] \cdot \Pr\left[V(\phi) \geq D\right] \\
&= \int_D^1 (z - D) \frac{\partial H(z;\phi)}{\partial z} dz
\end{aligned} \tag{5}
$$

We find that, for a fixed portfolio allocation ϕ, Eq. (5) is *decreasing* in the deposit rate D. We also find that, for a fixed D, Eq. (5) is *decreasing* in ϕ.

To prove these results, we need to consider three different cases based on the relationship between the values of D and ϕ. That's due to the piece-wise feature of the return distribution $H(z;\phi)$.

Case 1: $\mathbf{D} < \boldsymbol{\phi}$ Eq. (5) is given by

$$
\begin{aligned}
f_1 &\equiv \int_D^{\phi} (z - D) \frac{z}{(1-\phi)\phi} \, dz + \int_{\phi}^{1-\phi} (z - D) \frac{1}{1-\phi} \, dz + \int_{1-\phi}^{1} (z - D) \frac{1-z}{(1-\phi)\phi} \, dz \\
&= \frac{D^3}{6\phi - 6\phi^2} - D + \frac{1}{2}
\end{aligned}
$$

Note that $D \leq \frac{1}{2}$. The derivative with respect to D delivers

$$
\frac{\partial}{\partial D} f_1 = \frac{D^2}{2\phi - 2\phi^2} - 1 < 0
$$

which holds for $D < \phi \leq \frac{1}{2}$.

The derivative with respect to ϕ yields

$$
\frac{\partial}{\partial \phi} f_1 = \frac{D^3(2\phi - 1)}{6(\phi - 1)^2\phi^2} < 0
$$

which holds for $D < \phi \leq \frac{1}{2}$.

Hence the expected payoff when $\phi > D$ is decreasing in ϕ for a fixed D, and decreasing in D for a fixed ϕ.

Case 2: $\boldsymbol{\phi \leq D \leq 1-\phi}$ Eq. (5) is given by

$$f_2 \equiv \int_d^{1-\phi} (z-D)\frac{1}{1-\phi}\,dz + \int_{1-\phi}^{1} (z-D)\frac{1-z}{(1-\phi)\phi}\,dz$$
$$= \frac{3(D-1)\phi + 3(D-1)^2 + \phi^2}{6-6\phi}$$

Note that $\phi \leq D \quad \& \quad \phi \leq 1-D$.

The derivative with respect to D delivers

$$\frac{\partial}{\partial D} f_2 = -\frac{2D+\phi-2}{2(\phi-1)} < 0$$

which holds for $\phi < \frac{1}{2}$ and $\phi \leq D \leq 1-\phi$.

The derivative with respect to ϕ yields

$$\frac{\partial}{\partial \phi} f_2 = \frac{3(D-1)D - (\phi-2)\phi}{6(\phi-1)^2} < 0$$

The derivative changes signs at the threshold $\phi_T(D) = 1 - \sqrt{3D^2 - 3D + 1}$. If $\phi > \phi_T$, $\frac{d\Psi}{d\phi}$ is positive, while if $\phi < \phi_T$, $\frac{d\Psi}{d\phi}$ is negative.

Note that $\phi_T > D$ if $D < \frac{1}{2}$ and $\phi_T > 1-D$ if $D > \frac{1}{2}$. Since $\phi \leq D$, if $D < \frac{1}{2}$ then $\phi < \phi_T$. Also, since $\phi \leq 1-D$, if $D > \frac{1}{2}$ then $\phi < \phi_T$. Thus, the derivative with respect to ϕ is negative.

Hence the expected payoff when $\phi \leq D \leq 1-\phi$ is decreasing in ϕ for a fixed D, and decreasing in D for a fixed ϕ.

Case 3: $\boldsymbol{D > 1-\phi}$ Eq. (5) is given by

$$f_3 \equiv \int_d^1 (z-D)\frac{1-z}{(1-\phi)\phi}\,dz$$
$$= \frac{(D-1)^3}{6(\phi-1)\phi}$$

Note that $D \geq \frac{1}{2}$. The derivative with respect to D delivers

$$\frac{\partial}{\partial D} f_3 = -\frac{(1-D)^2}{2(1-\phi)\phi} < 0$$

which holds for $D < 1$ and $\phi \leq \frac{1}{2}$.

The derivative with respect to ϕ yields

$$\frac{\partial}{\partial \phi} f_3 = -\frac{(1-D)^3(1-2\phi)}{6(1-\phi)^2\phi^2} < 0$$

which also holds for $D < 1$ and $\phi \leq \frac{1}{2}$.

Thus the expected payoff when $D < 1-\phi$ is decreasing in ϕ for a fixed D, and decreasing in D for a fixed ϕ.

All Cases: We conclude from Cases 1, 2 and 3 together that the bank's expected payoff is decreasing in D for a fixed ϕ.

Characterization of Transparent Portfolio Equilibrium

The following two lemmas characterize the equilibrium when the bank holds a transparent portfolio. Namely, they specify the bank's optimal deposit rate $D^*(r,\phi)$, depositors' optimal withdraw probability $\omega^*(r,\phi)$, and the bank's optimal level of diversification $\phi^*(r)$.

We use these intermediate results in the proofs of the propositions in the main text.

Lemma A.1 (Equilibrium D and ω)**.** *Suppose the bank holds a transparent portfolio. For any outside value $r \in \left(0, \frac{1}{2}\right)$ and any portfolio allocation $(\phi, (1-\phi))$, bank's optimal deposit* *rate, D^*, as well as depositors' withdrawal probability ω^*, are as follows:*

1. $r < \frac{1}{4}$.

   (a) $\phi \leq \frac{2r+2}{5}$, *then* $D^* = \frac{1}{2}(r+1) - \frac{1}{4}\phi \in [\phi, (1-\phi)]$ *and* $\omega^* = \frac{1}{1-\phi}\left(\frac{1}{2}(r+1) - \frac{3}{4}\phi\right)$.

   (b) $\phi > \frac{2r+2}{5}$, *then* $D^* = \frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi} \in (r, \phi)$ *and* $\omega^* = \frac{\left(\frac{1}{3}r+\frac{1}{3}\sqrt{r^2-6\phi^2+6\phi}\right)^2}{2\phi(1-\phi)}$.

2. $r \geq \frac{1}{4}$.

   (a) $\phi \leq \frac{2}{3}(1-r)$, *then* $D^* = \frac{1}{2}(r+1) - \frac{1}{4}\phi \in [\phi, (1-\phi)]$ *and* $\omega^* = \frac{1}{1-\phi}\left(\frac{1}{2}(r+1) - \frac{3}{4}\phi\right)$.

   (b) $\phi > \frac{2}{3}(1-r)$, *then* $D^* = \frac{2}{3}r + \frac{1}{3} \in ((1-\phi), 1)$ *and* $\omega^* = \left(1 - \frac{\left(\frac{2}{3}-\frac{2}{3}r\right)^2}{2\phi(1-\phi)}\right)$.

Proof. Since $\phi \leq \frac{1}{2}$, then $\phi \leq 1 - \phi$. Thus there are three relevant cases.

Case 1: $D < \phi$. The expected payoff of each depositors is given by

$$W_B = D\left(1 - \frac{D^2}{2\phi(1-\phi)}\right) + r\frac{D^2}{2\phi(1-\phi)}.$$

The first order condition with respect to D yields

$$\frac{1}{2\phi(\phi-1)}\left(3D^2 - 2rD - 2\phi(1-\phi)\right) = 0,$$

which has the following solutions

$$\begin{aligned} D_{1a} &= \frac{1}{3}r - \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi} < 0 \\ D_{1b} &= \frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi} > 0 \end{aligned}$$

For D to be a feasible solutions, it must verify that $r < D < \phi$. We have that

$$\phi > \frac{2r+2}{5} \Rightarrow D_{1b} < \phi.$$

At the same time, $D_{1b} < \phi$ is sufficient to imply that $D_{1b} > r$, as

$$\phi > \frac{2r+2}{5} \Rightarrow \phi > \frac{1}{2} - \frac{1}{2}\sqrt{1-2r^2} \Rightarrow D > r$$

Case 2: $D > (1-\phi)$. The expected payoff of each depositors is given by

$$W_I = D\left(1 - \left(1 - \frac{(1-D)^2}{2\phi(1-\phi)}\right)\right) + r\left(1 - \frac{(1-D)^2}{2\phi(1-\phi)}\right).$$

The first order condition with respect to D yields

$$\frac{1}{2\phi\left(\phi-1\right)}\left(D-1\right)\left(2r-3D+1\right) = 0,$$

which admits the following solution

$$D_2 = \frac{2}{3}r + \frac{1}{3}.$$

For D to be a feasible solutions, it is sufficient that it verify that $D > (1-\phi)$, as $(1-\phi) > r$. We have that

$$D_2 > (1-\phi) \Longleftrightarrow \phi > \frac{2}{3}\left(1-r\right)$$

Case 3: $D \in [\phi, (1-\phi)]$. The expected payoff of each depositors is given by

$$W_I = D\left(1 - \frac{1}{1-\phi}\left(D - \frac{\phi}{2}\right)\right) + r\frac{1}{1-\phi}\left(D - \frac{\phi}{2}\right).$$

The first order condition with respect to D yields

$$\frac{1}{2\left(\phi-1\right)}\left(\phi - 2r + 4D - 2\right) = 0,$$

which admits the following solution

$$D_3 = \frac{1}{2}(r+1) - \frac{1}{4}\phi.$$

For D to be a feasible solutions, it is sufficient that it verify that $D_3 \leq (1-\phi)$, $D_3 \geq \phi$, and $D_3 > r$. We have that

$$D_3 \geq \phi \Leftrightarrow \frac{2}{5}(r+1) \geq \phi$$

and

$$D_3 \leq 1-\phi \Leftrightarrow \frac{2}{3}(1-r) \geq \phi.$$

Note that since $2(1-r) > \phi$, it follows that $D_3 > r$ as well.

□

Lemma A.2 (Equilibrium ϕ)**.** *Suppose that the bank holds a transparent portfolio. There exists a value* $\bar{r}_T = \frac{7}{18}$ *such that for any* $r < \bar{r}_T$*, the bank diversifies according to* $\phi^*(r) = \min\left\{\frac{2+2r}{5}, \frac{2-2r}{3}\right\}$*. If* $r \geq \bar{r}_T$*, the bank does not diversify* $\phi^* = 0$*.*

Proof. We need to calculate the optimal ϕ^* for the bank. That is, ϕ^* that maximizes

$$W_B = \int_D^1 \left[(z-D)\frac{\partial H(z,\phi)}{\partial z}\right] dz.$$

As the optimal deposit rate $D^*(r,\phi)$ (Lemma A.1) changes depending on ϕ and r, we need to consider various cases.

Case 1: $D^* < \phi$. $r < \frac{1}{4}$ & $\phi > \frac{2r+2}{5}$. From Lemma A.1, we have that $D^* = \frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi}$. Substituting into the objective function of the social planner, we obtain that

$$W_B = \int_{D^*}^{\phi} \left[(z - D^*) \frac{\partial}{\partial z} \left(\frac{z^2}{2\phi(1-\phi)} \right) \right] dz + \int_{\phi}^{1-\phi} \left[(z - D^*) \frac{\partial}{\partial z} \left(\frac{1}{1-\phi} (z - \frac{\phi}{2}) \right) \right] dz + \\ + \int_{1-\phi}^{1} \left[(z - D^*) \frac{\partial}{\partial z} \left(1 - \frac{(1-z)^2}{2\phi(1-\phi)} \right) \right] dz$$

or

$$W_B = \frac{1}{2} - \left(\frac{1}{3} r + \frac{1}{3} \sqrt{r^2 - 6\phi^2 + 6\phi} \right) + \frac{1}{6} \frac{\left(\frac{1}{3} r + \frac{1}{3} \sqrt{r^2 - 6\phi^2 + 6\phi} \right)^3}{\phi (1-\phi)}$$

Taking the first order condition with respect to ϕ yields

$$\frac{\partial}{\partial \phi} W_B = \frac{2\phi - 1}{\sqrt{r^2 - 6\phi^2 + 6\phi}} \left(1 + \frac{1}{162\phi^2} \frac{\left(r + \sqrt{r^2 - 6\phi^2 + 6\phi} \right)^2}{(\phi - 1)^2} \left(r \sqrt{r^2 - 6\phi^2 + 6\phi} + 3\phi^2 - 3\phi + r^2 \right) \right)$$

which has a unique solution of $\phi = \frac{1}{2}$. Note however that the second order condition with respect to ϕ evaluated at $\phi = \frac{1}{2}$ yields

$$\begin{aligned} \left.\frac{\partial^2}{\partial \phi^2} W_B \right|_{\phi = \frac{1}{2}} &= \left. \frac{2r^2 + 3}{(r^2 - 6\phi^2 + 6\phi)^{\frac{3}{2}}} \right|_{\phi = \frac{1}{2}} + r \left. \left(\frac{\partial}{\partial \phi} \frac{(2\phi - 1)}{162\phi^2} \frac{\left(r + \sqrt{r^2 - 6\phi^2 + 6\phi} \right)^2}{(\phi - 1)^2} \right) \right|_{\phi = \frac{1}{2}} \\ &= \left. \frac{2r^2 + 3}{(r^2 - 6\phi^2 + 6\phi)^{\frac{3}{2}}} \right|_{\phi = \frac{1}{2}} + r \left. \left(\frac{\left(r + \sqrt{r^2 - 6\phi^2 + 6\phi} \right)^2}{(1 - \phi)} \frac{1}{81\phi^3} \right) \right|_{\phi = \frac{1}{2}} > 0 \end{aligned}$$

which implies that the solution $\phi = \frac{1}{2}$ of the first order condition would be a local minimum. Thus, to find the value of ϕ that maximizes bank's objective we need to compare the value of W_B evaluated at the corners. In particular, we need $W_B\left(\frac{2(1+r)}{5}, D^*, r\right)$ and $W_B\left(\frac{1}{2}, D^*, r\right)$. We show that

$$W_B \left(\frac{2(1+r)}{5}, D^*, r \right) - W_B \left(\frac{1}{2}, D^*, r \right) > 0.$$

Thus,

$$\phi^* = \frac{2(1+r)}{5}.$$

Notice that the bank's expected payoff is everywhere decreasing in this region. That's because for any $\phi < \frac{1}{2}$, $\frac{\partial}{\partial \phi} W_B < 0$. Thus this justify why the optimal ϕ^* is the lowest possible value (lower bound). Indeed, for any $r < \frac{1}{4}$ (which holds), $\frac{2(1+r)}{5} < \frac{1}{2}$.

Case 2: $D^* > 1 - \phi$. $r \geq \frac{1}{4}$ & $\phi > \frac{2}{3}(1-r)$. Lemma A.1, we have that $D^* = \frac{2}{3}r + \frac{1}{3}$. Substituting into the objective function of of bank, we obtain that

$$W_B = \int_{D^*}^{1} \left((z - D^*) \frac{\partial}{\partial z} \left(1 - \frac{(1-z)^2}{2\phi(1-\phi)} \right) \right) dz$$

or

$$W_B = \frac{1}{6\phi(\phi - 1)} \left(\frac{2}{3} r - \frac{2}{3} \right)^3$$

Taking the first order condition with respect to ϕ yields

$$\frac{\partial}{\partial \phi} W_B = \frac{4}{81\phi^2 (\phi - 1)^2} (2\phi - 1)(1-r)^3 \leq 0.$$

which has a unique solution of $\phi = \frac{1}{2}$. Note however that the second order condition with respect to ϕ evaluated at $\phi = \frac{1}{2}$ yields

$$\begin{aligned} \left.\frac{\partial^2}{\partial \phi^2} W_B \right|_{\phi=\frac{1}{2}} &= \left.\frac{8}{81\phi^3 (1-\phi)^3} (1-r)^3 \left(3\phi^2 - 3\phi + 1\right) \right|_{\phi = \frac{1}{2}} \\ &= \frac{1}{81}(-128)(r-1)^3 > 0 \end{aligned}$$

which implies that the solution $\phi = \frac{1}{2}$ of the first order condition would be a local minimum. Thus, to find the value of ϕ that maximizes bank's objective we need to compare the value of W_B evaluated at the corners. In particular, we need $W_B\left(\frac{2(1-r)}{3}, D^*, r\right)$ and $W_B\left(\frac{1}{2}, D^*, r\right)$.

We show that

$$W_B\left(\frac{2\left(1-r\right)}{3},D^*,r\right)-W_B\left(\frac{1}{2},D^*,r\right)>0.$$

Thus,

$$\phi^*=\frac{2\left(1-r\right)}{3}.$$

Note bank's expected payoff is always decreasing in this case.

Case 3: $D^* \in [\phi, 1-\phi]$. $r<\frac{1}{4}$ & $\phi\leq\frac{2r+2}{5}$, and $r\geq\frac{1}{4}$ & $\phi\leq\frac{2}{3}\left(1-r\right)$.

From Lemma A.1, we have that $D^*=\frac{1}{2}\left(r+1\right)-\frac{1}{4}\phi$. Substituting into the objective function of bank, we obtain that

$$W_B=\int_{D^*}^{1-\phi}\left[(z-D^*)\frac{\partial}{\partial z}\left(\frac{1}{1-\phi}(z-\frac{\phi}{2})\right)\right]dz+\int_{1-\phi}^{1}\left[(z-D^*)\frac{\partial}{\partial z}\left(1-\frac{(1-z)^2}{2\phi(1-\phi)}\right)\right]dz$$

or

$$W_B=\frac{1}{96\left(1-\phi\right)}\left(12\left(r-1\right)^2+12\phi\left(r-1\right)+7\phi^2\right)$$

Taking the first order condition with respect to ϕ yields

$$\frac{\partial}{\partial\phi}W_B=\frac{1}{96\left(\phi-1\right)^2}\left(12r^2-12r-7\phi^2+14\phi\right).$$

Note that the second order condition with respect to ϕ yields

$$\frac{\partial^2}{\partial\phi^2}W_B=\frac{1}{48\left(1-\phi\right)^3}\left(12r^2-12r+7\right)>0$$

which implies that any feasible solution of the first order condition would be a local minimum. Thus, to find the value of ϕ that maximizes bank i's objective we need to compare the value of W_B evaluated at the corners. In particular, for $r<\frac{1}{4}$ we need to compare $W_B\left(\frac{2r+2}{5},D^*,r\right)$ and $W_B\left(0,D^*,r\right)$, while for $r\geq\frac{1}{4}$ we need to compare $W_B\left(\frac{2(1-r)}{3},D^{*/T},r\right)$ and $W_B\left(0,D^*,r\right)$.

Consider first $r < \frac{1}{4}$. Then

$$W_B\left(\frac{2r+2}{5}, D^*, r\right) - W_B\left(0, D^*, r\right) = -\frac{1}{240r-360}\left(30r^3 + 7r^2 - 16r + 7\right) > 0.$$

Thus

$$\phi^* = \frac{2r+2}{5}.$$

Consider next $r \geq \frac{1}{4}$. Then

$$W_B\left(\frac{2\left(1-r\right)}{3}, D^*, r\right) - W_B\left(0, D^*, r\right) = -\frac{1}{72\left(2r+1\right)}\left(18r-7\right)\left(r-1\right)^2.$$

Thus, if $r < \bar{r}_T = \frac{7}{18}$,

$$\phi^* = \frac{2\left(1-r\right)}{3},$$

and, if $r > \bar{r}_T$,

$$\phi^* = 0.$$

Notice that the bank's objective is non-monotonic in ϕ. Setting the first order condition with respect to ϕ to 0 yields $\phi_0 = 1 - \frac{\sqrt{12r^2-12r+7}}{\sqrt{7}}$. For $\phi < \phi_0$, $\frac{\partial}{\partial \phi} W_B < 0$ and so bank's expected payoff is decreasing in ϕ. For $\phi > \phi_0$, $\frac{\partial}{\partial \phi} W_B > 0$ and so bank's expected payoff is increaing in ϕ.

However, both ϕ^* for $r < \frac{1}{4}$ and for $r \geq \frac{1}{4}$ are greater than ϕ_0. This corresponds to the region in which the payoff is increasing in ϕ, justifying why the optimal ϕ^* is the highest possible value (upper bound).

All Cases: The derivations of all cases together imply that for $r < \bar{r}_T$ bank's expected payoff is maximized at $\phi^*(r) = \min\left\{\frac{2r+2}{5}, \frac{2-2r}{3}\right\}$. More precisely, for $r < \frac{1}{4}$, $\phi^*(r) = \frac{2r+2}{5}$ (Cases 1 and 3); for $\frac{1}{4} \leq r \leq \bar{r}_T$, $\phi^*(r) = \frac{2(1-r)}{3}$ (Cases 2 and 3); and for $r > \bar{r}_T$, $\phi^* = 0$ (Case 3),

$$\phi^*(r) = \begin{cases} \dfrac{2r+2}{5} & 0 < r \leq \frac{1}{4} \\ \dfrac{2-2r}{3} & \frac{1}{4} < r < \bar{r}_T \\ 0 & r \geq \bar{r}_T \end{cases} \tag{A.3}$$

Note that for any portfolio allocation $(\phi, 1-\phi)$ that is an equilibrium, the portfolio allocation $(1-\phi, \phi)$ is also an equilibrium. This follows from the symmetry of the bank' portfolio return distribution in the full information case. We will work with $(1-\phi, \phi)$ to make it comparable with the analysis with incomplete information. Thus, bank's expected payoff is maximized at $\phi^* = \max\left\{\frac{3-2r}{5}, \frac{21+2r}{3}\right\}$ if $r \leq \bar{r}_T$ and at $\phi^* = 1$ if $r > \bar{r}_T$.

□

Proof of Proposition 1

The result in the proposition follows from the Lemma A.1 and Lemma A.2.

Lemma A.2 immediately implies that $\phi^*(r) > 0$ only for $r < \bar{r}_T$, and $\phi^* = 0$ otherwise. In other words, the bank diversifies ($\phi^* \neq \{0, 1\}$) if $r < \bar{r}_T$. This proves the second statement in the proposition.

We next need to prove the first statement in the proposition: for $r < \bar{r}_T$, $\phi^*(r)$ minimizes the deposit rate D. That is,

$$\phi^*(r) = \arg\min_{\phi} D^*(\phi, r)$$

We use the results in Lemma A.1 and Lemma A.2. Let $r < \frac{1}{4}$. First, consider when $\phi \leq \frac{2+2r}{5}$. In this case, $D^*(\phi, r) = \frac{1}{2}(r+1) - \frac{1}{4}\phi$ is decreasing in ϕ. Thus by choosing the upper bound $\phi^* = \frac{2+2r}{5}$ the bank minimizes D^*. When $\phi > \frac{2+2r}{5}$, the deposit rate is $D^*(\phi, r) = \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi} + \frac{r}{3}$ which is increasing in ϕ. Thus by choosing the lower bound $\phi^* = \frac{2+2r}{5}$ the bank minimizes D.

Next let $r \in [\frac{1}{4}, \bar{r}_T)$. Consider when $\phi \leq \frac{2-2r}{3}$. In this case, the deposit rate is again

$D^*(\phi, r) = \frac{1}{2}(r+1) - \frac{1}{4}\phi$. By choosing the upper bound $\phi^* = \frac{2-2r}{3}$ the bank minimizes D. If $\phi > \frac{2-2r}{3}$, the deposit rate is $D^*(r) = \frac{2r}{3} + \frac{1}{3}$ which does not depend on ϕ.

Thus, for any $r < \bar{r}_T$, the bank chooses a portfolio allocation ϕ^* that minimizes $D^*(\phi)$.

Characterization of Unobservable Portfolio Equilibrium

The following two lemmas characterize the equilibrium when the bank holds an unobservable portfolio. Namely, they specify the bank's optimal deposit rate $D^*(r, \phi)$, depositors' optimal withdraw probability $\omega^*(r, \phi)$, and the bank's optimal level of diversification $\phi^*(r)$.

We use these intermediate results in the proofs of the propositions in the main text.

Lemma A.3 (Optimal D and ω)**.** *Suppose the bank holds an unobservable portfolio. For any outside value $r \in \left(0, \frac{1}{2}\right)$ and any portfolio allocation $(\phi, (1-\phi))$, bank's optimal deposit rate, D^*, as well as depositors' withdrawal probability ω^*, are as follows:*

1. $r < 0.25$

   (a) *If $\phi \leq \frac{2}{5}$, then $D^* = \frac{2-\phi}{4} \in (\phi, 1-\phi)$ and $\omega^* = 0$*

   (b) *If $\phi \in \left(\frac{2}{5}, \frac{1}{2}\right]$ then $D^* = \sqrt{\frac{2}{3}}\sqrt{\phi(1-\phi)} \leq \phi$ and $\omega^* = 0$*

   (c) *Else, otherwise there is no D for which the depositors are willing to continue the bank, i.e $\omega^* = 1$.*

2. $r \in [0.25, \frac{4}{15})$

   (a) *If $\phi \in (\frac{2}{5}, \frac{1}{2})$, then $D^* = \sqrt{\frac{2}{3}}\sqrt{\phi(1-\phi)} \leq \phi$ and $\omega^* = 0$*

   (b) *Else, otherwise there is no D for which the depositors are willing to continue the bank, i.e $\omega^* = 1$.*

3. $r \in [\frac{4}{15}, \sqrt{\frac{2}{27}})$

   (a) *If $\phi \in \left(\frac{1}{2} - \frac{\sqrt{2-27r^2}}{2\sqrt{2}}, \frac{1}{2}\right)$, then $D^* = \sqrt{\frac{2}{3}}\sqrt{\phi(1-\phi)} \leq \phi$ and $\omega^* = 0$*

(b) *Else, otherwise there is no D for which the depositors are willing to continue the bank, i.e $\omega^* = 1$.*

4. *For $r \geq \sqrt{\frac{2}{27}}$ there is no D for which the depositors are willing to continue the bank, i.e $\omega^* = 1$.*

Proof. Since $\phi \leq \frac{1}{2}$, then $\phi \leq 1 - \phi$. Thus there are three relevant cases.

Case 1: $D < \phi$. The expected payoff of each investor is given by

$$W_I = D\left(1 - \frac{D^2}{2\phi(1-\phi)}\right).$$

The first order condition with respect to D yields

$$\frac{\partial}{\partial D}\left(D\left(1 - \frac{D^2}{2\phi(1-\phi)}\right)\right) = -\frac{1}{2\phi\left(1-\phi\right)}\left(2\phi^2 - 2\phi + 3D^2\right) = 0$$

which has the following solutions

$$\begin{aligned} D_{1a} &= \sqrt{\frac{2}{3}}\sqrt{\phi\left(1-\phi\right)} > 0 \\ D_{1b} &= -\sqrt{\frac{2}{3}}\sqrt{\phi\left(1-\phi\right)} < 0 \end{aligned}$$

Second order consdition implies

$$\frac{\partial}{\partial D}\left(-\frac{1}{2\phi\left(1-\phi\right)}\left(2\phi^2 - 2\phi + 3D^2\right)\right) = -\frac{3}{\phi}\frac{D}{1-\phi} < 0$$

For D to be a feasible solutions, it must verify that $D < \phi$ and $D(1 - H(D, \phi)) > r$. We have that

$$\begin{aligned} D_{1a} &\leq \phi \Leftrightarrow \frac{2}{3}\phi\left(1-\phi\right) - \phi^2 = \frac{1}{3}\phi\left(2 - 5\phi\right) \leq 0 \\ \text{or } \phi &\geq \frac{2}{5}. \end{aligned}$$

At the same time

$$\begin{aligned} D_{1a}(1-H(D_{1a},\phi)) &> r \Longleftrightarrow \\ \frac{2}{3}\sqrt{\frac{2}{3}}\sqrt{\phi\,(1-\phi)} &> r \end{aligned}$$

which is true for any $\phi \in \left(\frac{1}{2}-\frac{1}{4}\sqrt{2}\sqrt{2-27r^2}, \frac{1}{4}\sqrt{2}\sqrt{2-27r^2}+\frac{1}{2}\right)$ which are the positive roots of

$$\frac{8}{27}\phi\,(1-\phi)-r^2=0$$

for any $r<\sqrt{\frac{2}{27}}$. We have that $\frac{1}{4}\sqrt{2}\sqrt{2-27r^2}+\frac{1}{2}>\frac{1}{2}$ and

$$\frac{1}{2}-\frac{1}{4}\sqrt{2}\sqrt{2-27r^2}<\frac{2}{5} \text{ for } r<\frac{4}{15}.$$

Thus, if $r<\frac{4}{15}$ and $\phi\in\left(\frac{2}{5},\frac{1}{2}\right)$, then we have a solution as

$$D_{1a}=\sqrt{\frac{2}{3}}\sqrt{\phi\,(1-\phi)}.$$

If $r\in\left(\frac{4}{15},\sqrt{\frac{2}{27}}\right)$ and $\phi\in\left(\frac{1}{2}-\frac{1}{4}\sqrt{2}\sqrt{2-27r^2},\frac{1}{2}\right)$ then we have a solution as

$$D_{1a}=\sqrt{\frac{2}{3}}\sqrt{\phi\,(1-\phi)}.$$

If $r>\sqrt{\frac{2}{27}}$then

$$\frac{8}{27}\phi\,(1-\phi)-r^2<0$$

or

$$D_{1a}(1-H(D_{1a},\phi))<r$$

Since D_{1a} maximizes $D(1-H(D,\phi))$, then there is no D for which depositors are willing to continue the bank.

Case 2: $D \geq (1-\phi)$. The expected payoff of each investor is given by

$$W_I = D\left(1 - \left(1 - \frac{(1-D)^2}{2\phi(1-\phi)}\right)\right).$$

The first order condition with respect to D yields

$$-\frac{1}{2\phi(\phi-1)}\left(3D^2 - 4D + 1\right) = 0$$

which admits the following solution

$$D_{2a} = \frac{1}{3} \text{ and } D_{2b} = 1.$$

Note that D_{2b} is actually an inflexion point, and that W_I is decreasing between $\left(\frac{1}{3}, 1\right)$. For D_{2a} to be a feasible solutions, it is sufficient that it verify that $D > (1-\phi)$. However note that $(1-\phi) > \frac{1}{2}$. This implies that W_I achieves a maximum when $D^* = (1-\phi)$.

We need to verify

$$D\left(1 - \left(1 - \frac{(1-D)^2}{2\phi(1-\phi)}\right)\right) > r$$

or

$$(1-\phi)\left(1 - \left(1 - \frac{(1-(1-\phi))^2}{2\phi(1-\phi)}\right)\right) = \frac{1}{2}\phi > r \Longleftrightarrow \phi > 2r$$

If $r > \frac{1}{4}$, then this condition is violated. For $r < \frac{1}{4}$ note that

$$2r < \frac{1}{4}\sqrt{2}\sqrt{2 - 27r^2} + \frac{1}{2}$$

which implies that for $\phi \in \left(2r, \left(\frac{1}{4}\sqrt{2}\sqrt{2-27r^2} + \frac{1}{2}\right)\right)$ we have two candidate solutions

$$D_{2a} = (1-\phi)$$

and

$$D_{1a} = \sqrt{\frac{2}{3}}\sqrt{\phi\left(1-\phi\right)}$$

We need to compare the payoff of depositors

$$\begin{aligned} W_I\left(D_{1a}\right) &= \sqrt{\frac{2}{3}}\sqrt{\phi\left(1-\phi\right)}\left(1-\frac{\left(\sqrt{\frac{2}{3}}\sqrt{\phi\left(1-\phi\right)}\right)^2}{2\phi(1-\phi)}\right) \\ &= \frac{2}{3}\sqrt{\frac{2}{3}}\sqrt{\phi\left(1-\phi\right)} \end{aligned}$$

while

$$W_I\left(D_{2a}\right) = \frac{1}{2}\phi$$

$$\begin{aligned} W_I\left(D_{2a}\right) &< W_I\left(D_{1a}\right) \Longleftrightarrow \\ \frac{1}{2}\phi - \frac{2}{3}\sqrt{\frac{2}{3}}\sqrt{\phi\left(1-\phi\right)} &< 0 \end{aligned}$$

$$\left(\frac{1}{2}\phi\right)^2 - \left(\frac{2}{3}\sqrt{\frac{2}{3}}\sqrt{\phi\left(1-\phi\right)}\right)^2 = \frac{1}{4}\phi - \frac{8}{27}\left(1-\phi\right) = \frac{59}{108}\phi - \frac{32}{108} < 0 \Longleftrightarrow \phi < \frac{32}{59}$$

which is true, since $\phi < \frac{1}{2} < \frac{32}{59}$.

Case 3: $D \in [\phi, (1-\phi)]$. The expected payoff of each investor is given by

$$W_I = D\left(1-\frac{1}{1-\phi}\left(D-\frac{\phi}{2}\right)\right).$$

The first order condition with respect to D yields

$$\frac{1}{2\left(\phi-1\right)}\left(\phi+4D-2\right) = 0$$

which admits the following solution

$$D_3 = \frac{1}{4}(2-\phi).$$

For D to be a feasible solutions, it is sufficient that it verify that $D_3 \leq (1-\phi)$, $D_3 \geq \phi$, and $D_3(1-H(D_3,\phi)) > r$. We have that

$$\frac{1}{4}(2-\phi) > \phi \Leftrightarrow \frac{2}{5} > \phi$$

and

$$D_3 \leq 1-\phi \Leftrightarrow \frac{1}{4}(2-\phi) \leq 1-\phi \Leftrightarrow \phi < \frac{2}{3}$$

and

$$\begin{aligned} D\left(1-\frac{1}{1-\phi}\left(D-\frac{\phi}{2}\right)\right) &> r \Leftrightarrow \\ \frac{1}{4}(2-\phi)\left(1-\frac{1}{1-\phi}\left(\frac{1}{4}(2-\phi)-\frac{\phi}{2}\right)\right) &= -\frac{(\phi-2)^2}{16\phi-16} > r \end{aligned}$$

which is always true for $r < \frac{1}{4}$. If $r > \frac{1}{4}$, there is no D for which the investor is willing to continue the bank.

Note again that $\phi \in \left(2r, \frac{2}{5}\right)$ we have two candidate solutions

$$D_{2a} = (1-\phi)$$

and

$$D_3 = \frac{1}{4}(2-\phi).$$

We need to compare the payoff of depositors

$$\begin{aligned} W_I(D_3) &= \frac{1}{4}(2-\phi)\left(1-\frac{1}{1-\phi}\left(\frac{1}{4}(2-\phi)-\frac{\phi}{2}\right)\right) \\ &= \frac{(\phi-2)^2}{16(1-\phi)} \end{aligned}$$

while

$$W_I(D_{2a}) = \frac{1}{2}\phi$$

$$\begin{aligned} W_I(D_{2a}) &< W_I(D_3) \Longleftrightarrow \\ \frac{(\phi-2)^2}{16(1-\phi)} - \frac{1}{2}\phi &= \frac{1}{16(1-\phi)}(3\phi-2)^2 > 0 \end{aligned}$$

Thus, for $\phi < \frac{2}{5}$, the solution is

$$D_3 = \frac{1}{4}(2-\phi).$$

□

Lemma A.4 (Optimal ϕ)**.** *Suppose the bank holds an unobservable portfolio. There exists* $\bar{r}_{U_{low}} = \frac{4}{15}$ *and* $\bar{r}_U = \sqrt{\frac{2}{27}}$ *such that if* $r < \bar{r}_{U_{low}}$*, then* $\phi^* = \frac{2}{5}$*. If* $\bar{r}_{U_{low}} \leq r < \bar{r}_U$*, then* $\phi^* = \frac{1}{2} - \frac{\sqrt{2}}{4}\sqrt{2-27r^2}$*. If* $r \geq \bar{r}_U$*, then* ϕ^* *is undetermined.*

Proof. We need to calculate the optimal ϕ^* for the bank. That is, ϕ^* that maximizes

$$W_B = \int_D^1 \left[(z-D)\frac{\partial H(z,\phi)}{\partial z}\right] dz.$$

As the optimal deposit rate $D^*(r,\phi)$ (Lemma A.3) changes depending on ϕ and r, we need to consider various cases.

Case 1a $r < 0.25$ and $\phi \leq \frac{2}{5}$

From Lemma A.3, we have that $D^* = \frac{(2-\phi)}{4} \in (\phi, 1-\phi)$. Substituting into the objective function of bank, we obtain that

$$W_B = \int_{D^*}^{1-\phi} \left[(z - D^*) \frac{\partial}{\partial z} \left(\frac{1}{1-\phi} (z - \frac{\phi}{2}) \right) \right] dz + \int_{1-\phi}^{1} \left[(z - D^*) \frac{\partial}{\partial z} \left(1 - \frac{(1-z)^2}{2\phi(1-\phi)} \right) \right] dz$$

or

$$W_B = \frac{12 - 12\phi + 7\phi^2}{96(1-\phi)}$$

Taking the first order condition with respect to ϕ yields

$$\frac{\partial}{\partial \phi} W_B = -\frac{7(\phi - 2)\phi}{96(\phi - 1)^2} > 0$$

which implies that the payoff of the bank is always increasing, and hence bank chooses

$$\phi^* = \frac{2}{5}$$

This implies

$$D^* = \frac{2}{5}$$

and it holds that $D^* \geq \phi^*$.

Note that the expected payoff of the depositors is

$$D^* \left(1 - \frac{1}{1-\phi^*} (D^* - \frac{\phi^*}{2}) \right) = \frac{4}{15} > r \quad \forall r < 0.25$$

and bank's payoff is

$$W_B(\phi^*) = \frac{13}{10}$$

Case 1b $r < 0.25$ and $\phi \in (\frac{2}{5}, \frac{1}{2}]$

From Lemma A.3, we have that $D^* = \sqrt{\frac{2}{3}}\sqrt{\phi(1-\phi)} \leq \phi$. Substituting into the objective function of bank, we obtain that

$$W_B = \int_{D^*}^{\phi} \left[(z - D^*)\frac{\partial}{\partial z}\left(\frac{z^2}{2\phi(1-\phi)}\right)\right] dz + \int_{\phi}^{1-\phi} \left[(z - D^*)\frac{\partial}{\partial z}\left(\frac{1}{1-\phi}(z - \frac{\phi}{2})\right)\right] dz$$
$$+ \int_{1-\phi}^{1} \left[(z - D^*)\frac{\partial}{\partial z}\left(1 - \frac{(1-z)^2}{2\phi(1-\phi)}\right)\right] dz$$

or

$$W_B = \frac{1}{2} - \frac{8}{9}\sqrt{\frac{2}{3}}\sqrt{(1-\phi)\phi}$$

Taking the first order condition with respect to ϕ yields

$$\frac{\partial}{\partial \phi} W_B = \frac{4\sqrt{\frac{2}{3}}(2\phi - 1)}{9\sqrt{(1-\phi)\phi}}$$

Note that the second order condition with respect to ϕ yields

$$\frac{\partial^2}{\partial^2 \phi} W_B = \frac{2\sqrt{\frac{2}{3}}}{9(-((\phi-1)\phi))^{3/2}} > 0$$

which implies that any feasible solution of the first order condition would be a local minimum. Thus, to find the value of ϕ that maximizes bank's objective we need to compare the value of W_B evaluated at the corners. In particular, we compare

$$W_B\left(\frac{2}{5}\right) - W_B\left(\frac{1}{2}\right) = 0.144444 - 0.137113 > 0$$

Hence bank chooses

$$\phi^* = \frac{2}{5}$$

This implies

$$D^* = \frac{2}{5}$$

which is the same solution as Case 1a.

Note that the expected payoff of the depositors is

$$D^* \left(1 - \frac{(D^*)^2}{2\phi^*(1-\phi^*)}\right) = \frac{4}{15} > r \quad \forall r < 0.25$$

and bank's payoff is

$$W_B(\phi^*) = \frac{13}{10}$$

Case 2 $r \in [0.25, \frac{4}{15}]$ and $\phi \in (\frac{2}{5}, \frac{1}{2})$

This is the same as Case 1b:

$$\phi^* = \frac{2}{5}$$
$$D^* = \frac{2}{5}$$

Case 3 $r \in [\frac{4}{15}, \frac{\sqrt{2}}{\sqrt{27}})$ and $\phi \in (1 - \frac{1}{4}\sqrt{2}\sqrt{2 - 27r^2}, \frac{1}{2})$

From Lemma A.3, we have that $D^* = \sqrt{\frac{2}{3}}\sqrt{\phi(1-\phi)} \leq \phi$. This is the same as Case 1b with different intervals for r and ϕ.

We know that any feasible solution of the first order condition would be a local minimum. Thus, to find the value of ϕ that maximizes bank's objective we need to compare the value of W_B evaluated at the corners. In particular, we compare

$$W_B\left(\frac{1}{2}\right) - W_B\left(1 - \frac{1}{4}\sqrt{2}\sqrt{2 - 27r^2}\right) = 0.137113 - 0.16667\left(3 - 8\sqrt{r^2}\right)$$

Note that $W_B\left(1 - \frac{1}{4}\sqrt{2}\sqrt{2 - 27r^2}\right)$ is decreasing in r, and at the r threshold values we have

$$W_B\left(1-\frac{1}{4}\sqrt{2}\sqrt{2-27\cdot\left(\frac{4}{15}\right)^2}\right)=0.144444$$

$$W_B\left(1-\frac{1}{4}\sqrt{2}\sqrt{2-27\cdot\left(\frac{\sqrt{2}}{\sqrt{27}}\right)^2}\right)=0.137113$$

Hence for any $r<\frac{\sqrt{2}}{\sqrt{27}}$, $W_B\left(\frac{1}{2}\right)-W_B\left(1-\frac{1}{4}\sqrt{2}\sqrt{2-27r^2}\right)<0$ and bank chooses

$$\phi^*=1-\frac{1}{4}\sqrt{2}\sqrt{2-27r^2}$$

This implies

$$D^*=\frac{3r}{2}$$

Note that the expected payoff of the depositors is

$$D^*\left(1-\frac{(D^*)^2}{2\phi^*(1-\phi^*)}\right)=r$$

and the bank's payoff is

$$W_B(\phi^*)=\frac{1}{6}(3-8r)$$

□

Proof of Proposition 2

The proposition follows from the results in Lemma A.3 and Lemma A.4.

Let $r<\frac{1}{4}$. First, consider when $\phi\leq\frac{2}{5}$ (Case 1a). The optimal debt rate $D^*=\frac{2-\phi}{4}\in(\phi,1-\phi)$ is decreasing in ϕ. By choosing the upper bound $\phi^*=\frac{2}{5}$ the bank minimizes the deposit rate D^*. Second, consider when $\phi \in (\frac{2}{5}, \frac{1}{2}]$ (Case 1b). The optimal debt rate $D^* = \sqrt{\frac{2}{3}}\sqrt{\phi(1-\phi)}$ is increasing in ϕ. Hence, by choosing the lower bound $\phi^* = \frac{2}{5}$ the bank minimizes the deposit rate D.

Next, let $r \in (\frac{1}{4}, \bar{r}_{U_{low}}]$ and $\phi \in (\frac{2}{5}, \frac{1}{2})$ (Case 2). In this case, the optimal debt rate is also $D^* = \sqrt{\frac{2}{3}}\sqrt{\phi(1-\phi)}$. By choosing the lower bound $\phi^* = \frac{2}{5}$ the bank minimizes the deposit rate D.

Lastly, let $r \in [\bar{r}_{U_{low}}, \bar{r}_U)$ and $\phi \in (1 - \frac{1}{4}\sqrt{2}\sqrt{2-27r^2}, \frac{1}{2})$ (Case 3). In this case, the optimal debt rate is also $D^* = \sqrt{\frac{2}{3}}\sqrt{\phi(1-\phi)}$. By choosing the lower bound $\phi^* = 1 - \frac{1}{4}\sqrt{2}\sqrt{2-27r^2}$ the bank minimizes the deposit rate D.

Thus, for any $r < \bar{r}_U$, the bank chooses a portfolio allocation ϕ^* that minimizes $D^*(\phi)$. This proves the first statement in the proposition.

The second statement in the proposition is straightforward. If $r > \bar{r}_U$, there exists no deposit rate D for which the probability of withdrawal ω^* is not equal to 1. Hence, ϕ^* is indeterminate. In other words, $\omega^* = 0$ whenever there exists a deposit rate $D^* > 0$.

Characterization of Opaque Portfolio Equilibrium

The following two lemmas characterize the equilibrium when the bank holds an opaque portfolio. Namely, they specify the bank's optimal deposit rate $D^*(r, \phi)$, depositors' optimal withdraw probability $\omega^*(r, \phi)$, and the bank's optimal level of diversification $\phi^*(r)$.

We use these intermediate results in the proofs of the propositions in the main text.

Lemma A.5 (Depositors Withdrawal Decision)**.** *Suppose the bank holds an opaque portfolio and depositors receive the interim signal R_1. For each portfolio allocation, $\phi \in (0, 1)$, and each deposit rate, D, set at date 0, the optimal withdraw decision of depositors ω^* is characterized by a threshold strategy*

$$\omega^*(\mathbf{s}) = \begin{cases} 1 & \text{if } R_1 < \bar{\omega} \\ 0 & \text{if } R_1 \geq \bar{\omega} \end{cases}, \tag{A.4}$$

where

$$\bar{\omega} = \max\left\{\min\left\{1, \frac{D}{\phi} - \frac{1-\phi}{\phi}G^{-1}\left(1-\frac{r}{D}\right)\right\}, 0\right\}. \tag{A.5}$$

Proof. With an opaque portfolio, the depositors can partially infer the bank's portfolio return as they observe $\mathbf{s} = (R_1, \varnothing)$. They continue funding the bank if the amount expected to receive at date 2 conditional on their signal $\mathbf{s}$, $D \cdot \Pr\left(D \leq V\left(\phi\right)|\mathbf{s}\right)$, is larger than the reservation value r, which they obtain when they liquidate the bank at date 1. If the realization of R_1 is sufficiently high so that $D < \phi R_1$, then the depositors get repaid D with certainty. However, if $D \geq \phi R_1$, then depositors get repaid D only if $R_2 \geq \frac{D-\phi R_1}{1-\phi}$, which occurs with probability $\left(1 - G\left(\frac{D-\phi R_1}{1-\phi}\right)\right)$. Otherwise, the bank goes into default at date 2, and the depositors get 0. Thus, everything else equal, the higher the face value of debt is, the lower the probability that the depositors get repaid when $D \geq \phi R_1$.

This implies that depositors find it optimal to continue funding the bank when

$$\Pr\left(R_2 \geq \max\left\{\frac{D-\phi R_1}{1-\phi}, 0\right\}\middle| R_1\right) \geq \frac{r}{D},$$

or when their signal is sufficiently large. Isolating R_1 we get the lemma.

□

Lemma A.6 (Depositors' Expected Payoff)**.** *Suppose the bank holds an opaque portfolio and depositors receive the interim signal R_1. The optimal deposit rate D^* is characterzied as follows:*

1. *If $D \leq \phi$, D^* satisfies*

$$\int_{\bar{\omega}}^{\frac{D}{\phi}} \left(1 - \frac{D-\phi R_1}{1-\phi}\right) dR_1 + \int_{\frac{D}{\phi}}^{1} 1 dR_1 + D\int_{\bar{\omega}}^{\frac{D}{\phi}} \frac{\partial}{\partial D}\left(1 - \frac{D-\phi R_1}{1-\phi}\right) dR_1 + D\int_{\bar{\omega}}^{\frac{D}{\phi}} \frac{\partial}{\partial D} 1 dR_1 = 0. \tag{A.6}$$

2. *If $D \leq \phi$, D^* satisfies*

$$\int_{\bar{\omega}}^{1}\left(1-\frac{D-\phi R_1}{1-\phi}\right)dR_1+D\int_{\bar{\omega}}^{1}\frac{\partial}{\partial D}\left(1-\frac{D-\phi R_1}{1-\phi}\right)dR_1=0. \tag{A.7}$$

Proof. Depositors' expected payoff is given by

$$W_I=D\cdot\mathbb{E}_{R_1}\left(\Pr\left(R_2\geq\frac{D^*-\phi R_1}{1-\phi}\right)\middle|\,R_1\geq\bar{\omega}\right)\Pr\left(R_1\geq\bar{\omega}\right)+r\cdot\Pr\left(R_1<\bar{\omega}\right). \tag{A.8}$$

Making use of Lemma A.5 and taking the first order conition of A.8 with respect to D we get,

$$\frac{1}{D}=-\frac{\frac{\partial}{\partial D}\mathbb{E}_{R_1}\left(\Pr\left(R_2\geq\frac{D^*-\phi R_1}{1-\phi}\right)\middle|\,R_1\geq\bar{\omega}\right)\Pr\left(R_1\geq\bar{\omega}\right)}{\mathbb{E}_{R_1}\left(\Pr\left(R_2\geq\frac{D^*-\phi R_1}{1-\phi}\right)\middle|\,R_1\geq\bar{\omega}\right)\Pr\left(R_1\geq\bar{\omega}\right)}. \tag{A.9}$$

We need to distinguish two main cases when $\bar{\omega}<1$: $D^*\leq\phi$ and $D^*>\phi$.

In the first case, when in equilibrium $D^*\leq\phi$, the first order condition (A.9) becomes

$$\int_{\bar{\omega}}^{\frac{D}{\phi}}\left(1-\frac{D-\phi R_1}{1-\phi}\right)dR_1+\int_{\frac{D}{\phi}}^{1}1dR_1+D\int_{\bar{\omega}}^{\frac{D}{\phi}}\frac{\partial}{\partial D}\left(1-\frac{D-\phi R_1}{1-\phi}\right)dR_1+D\int_{\bar{\omega}}^{\frac{D}{\phi}}\frac{\partial}{\partial D}1dR_1=0.$$

Integrating, we obtain that the deposit rate must satisfy the following equation

$$\left(D^*-\bar{\omega}\phi\right)\left(3D^*-\bar{\omega}\phi\right)=2\phi\left(1-\phi\right)\left(1-\bar{\omega}\right),$$

for any $\phi\in(0,1)$.

In the second case, when in equilibrium $D^*>\phi$, the first order condition (A.9) becomes

$$\int_{\bar{\omega}}^{1}\left(1-\frac{D-\phi R_1}{1-\phi}\right)dR_1+D\int_{\bar{\omega}}^{1}\frac{\partial}{\partial D}\left(1-\frac{D-\phi R_1}{1-\phi}\right)dR_1=0.$$

Integrating, we obtain that the deposit rate must satisfy the following equation

$$D^* = \frac{1}{4}\phi\bar{\omega} - \frac{1}{4}\phi + \frac{1}{2},$$

for any $\phi \in (0,1)$.

$\square$

Lemma A.7 (Equilibrium D and ω)**.** *Suppose the bank holds an opaque portfolio. For any outside value $r \in \left(0, \frac{1}{2}\right)$ and any portfolio allocation $(\phi, (1-\phi))$, bank's optimal deposit rate, D^*, as well as depositors' withdrawal probability ω^*, are as follows:*

1. *if $0 \leq \phi \leq \min\{\frac{2}{5}, \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)\}$, then $D^* = \frac{2-\phi}{4}$ and $\bar{\omega} = 0$.*
2. *if $\frac{2}{5} \leq \phi \leq \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$, then $D^* = \sqrt{\frac{2}{3}\phi(1-\phi)}$ and $\bar{\omega} = 0$.*
3. *if $\max\{\frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right), 1 - \frac{1}{4r}\} < \phi \leq \frac{1}{6}\left(1 - r + \sqrt{r(r+10) + 1}\right)$ then $D^* = \frac{1}{6}\left(1 + \sqrt{1 + 12r(1-\phi)}\right)$ and $\bar{\omega} = \frac{1}{3\phi}\left(-4 + 3\phi + \sqrt{1 + 12r(1-\phi)}\right)$, $0 < \bar{\omega} < 1$.*
4. *if $\max\{\frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right), \frac{1}{6}\left(1 - r + \sqrt{r(r+10) + 1}\right)\} < \phi \leq 1$, then D^* is the largest root of equation*

   $$-4D^3 + D^2(2r + \phi + 1) + r^2(\phi - 1) = 0,$$

   and $\bar{\omega} = \frac{1}{\phi}\left(D^* - (1-\phi)(1 - \frac{r}{D^*})\right)$, $0 < \bar{\omega} < 1$.
5. *if $0 \leq \phi < 1 - \frac{1}{4r}$, then $\bar{\omega} = 1$.*

Proof. Start from equations (A.5) and (A.9) in Lemma A.5 and Lemma A.6, respectively. Let $G \sim U[0,1]$. We need to consider $\bar{\omega} = 0$, $\bar{\omega} = 1$ and $0 < \bar{\omega} < 1$ separately. Moreover, we need to differentiate the following cases: $D^* < \phi$, and $D^* \geq \phi$.

1. No early withdrawal, $\bar{\omega} = 0$.

1. $\frac{D^*}{\phi} > 1$. In this case, depositors payoff simplifies to

$$W = D \int_0^1 \left(1 - \frac{D - \phi z}{1 - \phi}\right) dz.$$

The first order condition is

$$\frac{4D + \phi - 2}{2(\phi - 1)\phi} = 0,$$

which implies

$$D^* = \frac{1}{4}(2 - \phi).$$

The second order condition holds (SOC< 0), thus the above D^* is a maximum. $D^* > \phi$ requires $0 \leq \phi \leq \frac{2}{5}$, while $\bar{\omega} = 0$ requires $\phi \leq \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$, which leads the first case.

2. $\frac{D^*}{\phi} < 1$. In this case, depositors payoff simplifies to

$$W = \int_0^{\frac{D}{\phi}} \left(1 - \frac{D - \phi z}{1 - \phi}\right) dz + D(1 - \frac{D}{\phi})$$

The first order condition is

$$\frac{3D^2 + 2(\phi - 1)\phi}{2(\phi - 1)\phi} = 0,$$

which implies

$$D^* = \sqrt{\frac{2}{3}\phi(1 - \phi)}.$$

The second order condition holds (SOC< 0), thus the above D^* is a maximum. $D^* < \phi$ requires $\phi \geq \frac{2}{5}$, while $\bar{\omega} = 0$ requires $\phi \leq \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$, which leads the second case.

2. Some early withdraw, $0 < \bar{\omega} < 1$**.**

$$\bar{\omega} = \frac{D}{\phi} - \frac{1-\phi}{\phi}\left(1 - \frac{r}{D}\right)$$

We again separately consider two cases:

1. $\frac{D^*}{\phi} > 1$. In this case, depositors payoff simplifies to

$$W = D\int_{\bar{\omega}}^{1}\left(1 - \frac{D - \phi z}{1-\phi}\right)dz + r\bar{\omega}$$

Substituting for $\bar{\omega}$ and taking first order condition implies

$$\frac{(D(1-3D) + r(1-\phi))\,(r(1-\phi) - D(1-D))}{2\phi(1-\phi)D^2} = 0,$$

This is a quadratic equation with four roots: $D_1 = \frac{1}{6}\left(1 - \sqrt{1 + 12r(1-\phi)}\right) < 0$. $D_{2,4} = \frac{1}{2}\left(1 \pm \sqrt{4r\phi - 4r + 1}\right)$, and $\bar{\omega}(D_2) = \bar{\omega}(D_4) = 1$. Thus, the only relevant face value is $D_3 = \frac{1}{6}\left(1 + \sqrt{1 + 12r(1-\phi)}\right)$. Note that $\bar{\omega}(D) < 1$ only if $D_2 < D < D_4$, thus the optimal face value can be in this interval. Moreover, $D_2 < D_3 < D_4$.

Next, the second order condition is given by

$$\frac{\frac{r^2(1-\phi)^2}{D^3} + 3D - 2}{(1-\phi)\phi}$$

Letting SOC$= 0$ leads to a quadratic equation. Only two of the roots are in between D_2 and D_3, and $D_2 < D_1^{soc} < D_3 < D_2^{soc} < D_4$. Thus the second order condition changes sign twice on the interval $[D_2, D_4]$. Moreover, the second derivative evaluated at D_2 and D_4 is

$$\frac{1 \pm \left(\sqrt{1 - 4r(1-\phi)}\right)(1 - 4r(1-\phi))}{2r\phi(1-\phi)^2} > 0,$$

which implies that D_2 and D_4 are local minima, and that the second derivative is negative at D_3, thus D_3 is a (local) maximum. Since first order condition is positive between D_2 and D_3, and negative between D_3 and D_4, D_3 is the global maximum in the interval $[D_2, D_4]$. Thus we have

$$D^* = \frac{1}{6}\left(1 + \sqrt{1 + 12r(1-\phi)}\right)$$
$$\bar{\omega} = \frac{1}{3\phi}\left(-4 + 3\phi + \sqrt{1 + 12r(1-\phi)}\right).$$

Lastly, $D^* > \phi$ requires $\phi < \frac{1}{6}\left(1 - r + \sqrt{r^2 + 10r + 1}\right)$, $\bar{\omega} < 1$ requires $\phi > 1 - \frac{1}{4r}$ and $\bar{\omega} > 0$ requires $\phi > \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)$. This leads the third case. Moreover, $\bar{\omega}$ cannot exceed 1, thus $\bar{\omega} = 1$ if $\phi \geq 1 - \frac{1}{4r}$, which leads the fifth case.

2. $\frac{D^*}{\phi} < 1$. Here we need to consider two sub-cases

   (a) $\bar{\omega} < \frac{D^*}{\phi} \Leftrightarrow r < D^* < \phi$. In this case, depositors payoff simplifies to

$$W = \int_{\bar{\omega}}^{\frac{D}{\phi}} \left(1 - \frac{D - \phi z}{1 - \phi}\right) dz + D(1 - \frac{D}{\phi}) + r\bar{\omega},$$

   Substituting for $\bar{\omega}$ and taking the first order condition implies

$$\frac{-4D^3 + D^2(1 + \phi + 2r) - r^2(1-\phi)}{2D^2\phi} = 0.$$

   The numerator is a cubic function in D, with $\Delta = -432r^4(1-\phi)^2 + 4r^2(1-\phi)(1+\phi+2r)^3$, thus $\Delta < 0$ implies $(1+\phi+2r)^3 - 108r^2(1-\phi) < 0$. For any pair (r, ϕ) that satisfy $\Delta < 0$,

$$\phi < \tilde{\phi}(r) = \max\{\frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right), \frac{1}{6}\left(-r + \sqrt{r(r+10)+1} + 1\right)\},$$

   and (r, ϕ) is covered by one of the first 3 cases. Thus when $\phi > \tilde{\phi}(r)$, $Delta > 0$ and the cubic first order condition has 3 distinct real roots, $D_1 < D_2 < D_3$.

$D_1 < 0$, so it is not the solution. Moreover, note that the derivative of depositors surplus approaches $-\infty$ as $D \to 0$ from above, and as $D \to \infty$.

Next, the second order condition is given by

$$\frac{r^2(1-\phi) - 2D^3}{D^3\phi},$$

which has one root: $D^{soc} = \left(\frac{r^2(1-\phi)}{2}\right)^{\frac{1}{3}}$, and it is positive iff $D < D^{soc}$. Moreover, $D_2 < D^{soc} < D_3$, thus D_2 is a local minimum while D_3 is a local maximum. Thus either D_3 is the optimal face value, or the minimum feasible D, which in this case is $D = r$. Comparing the two values leads $W(\phi, r, D_3) > W(\phi, r, r)$, $\forall\ \phi > \tilde{\phi}(r)$. Thus we have

$$\begin{aligned} D^* &= D_3 \\ \bar{\omega} &= \frac{1}{\phi}\left(D^* - (1-\phi)\left(1 - \frac{r}{D^*}\right)\right). \end{aligned}$$

Lastly, note that if $D^* = r$, first order condition implies that $\phi = r$. However, $r < \tilde{\phi}(r)$, which in turn implies that whenever $\frac{D^*}{\phi} < 1$, $D^* > r$ and thus the next case is never relevant.

(b) $\bar{\omega} > \frac{D^*}{\phi} \Leftrightarrow D^* < \min\{r, \phi\}$. As argued above, this case does not arise in equilibrium.

Lastly, if $D^* = \phi$, first order condition implies $\phi = \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$, and $\bar{\omega} > 0$ implies $\phi > \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$. This leads to the forth case.

Always liquidate.

$$\frac{D^*}{\phi} > 1$$

□

Lemma A.8 (Equilibrium ϕ)**.** *Suppose the bank holds an opaque portfolio and depositors receive the interim signal R_1. There exists $\bar{r}_O \approx 0.477$ such that for any $r < \bar{r}_O$, the bank holds in equilibrium a diversified portfolio, $\phi^*(r) \in (0,1)$. If $r \geq \bar{r}_O$, the bank holds a concentrated portfolio and invests only in the transparent project, $\phi^*(r) = 1$.*

Proof. We need to calculate the optimal ϕ for the bank. That is, ϕ that maximizes

$$W_B = \int_{\bar{\omega}}^{1} \int_{\max\left\{\frac{D^* - \phi R_1}{1-\phi}, 0\right\}}^{1} \left(\phi R_1 + (1-\phi) R_2 - D^*(\phi)\right) dR_2 dR_1$$

As the optimal deposit rate $D^*(r,\phi)$ (Lemma A.7) changes depending on ϕ and r, we need to consider various cases.

Case 1 : $0 \leq \phi \leq \min\{\frac{2}{5}, \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)\}$.

Here $D^* = \frac{2-\phi}{4}$ and $\bar{\omega} = 0$. Substitute in the planner objective function to get

$$W = \frac{7\phi^2 + 12(1-\phi)}{96(1-\phi)}.$$

Since $\bar{\omega} = 0$, the optimal face value and bank profit are independent of r. Observe that $\frac{dW_B}{d\phi} > 0$ and $\frac{d^2 W_B}{d\phi^2} > 0$, thus the bank profit function is increasing and convex in this region. It follows that if the equilibrium level of opacity is in this region we will have

$$\phi^* = \min\left\{\frac{2}{5}, \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)\right\},$$

or

$$\phi^* = \begin{cases} \frac{2}{5} & \text{if } 0 < r < \bar{r}_{O_1} \\ \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right) & \text{if } \bar{r}_{O_1} < r < \frac{1}{4} \end{cases}$$

where $\bar{r}_{O_1} = \frac{2}{15}$.

Case 2 : $\frac{2}{5} \leq \phi \leq \frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right)$.

Here $D^* = \sqrt{\frac{2}{3}\phi(1-\phi)}$ and $\bar{\omega} = 0$. Substitute in the bank objective function to get

$$W_B = \frac{1}{2} - \frac{8}{9}\sqrt{\frac{2}{3}\left(1-\phi\right)\phi}.$$

Since $\bar{\omega} = 0$, again the optimal face value and bank profit are independent of r. Observe that $\frac{dW_i}{d\phi} = 0$ at $\phi = \frac{1}{2}$ and $\frac{d^2W}{d\phi^2} > 0$, thus the objective function is convex, and the maximum is attained on one of the corners, i.e. $\phi^* = \frac{2}{5}$ or $\phi^* = \frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right)$. Direct comparison of the bank profit on the two boundaries reveals that $W(\frac{2}{5}) \geq W\left(\frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right)\right)$ for $0 < r < \bar{r}_{O_1}$. It follows that if the equilibrium level of opacity is in this region we will have

$$\phi^* = \frac{2}{5}.$$

Case 3 : $\max\left\{\frac{2}{3}\left(2(1-2r)-\sqrt{(4r)^2-4r+1}\right), 1-\frac{1}{4r}\right\} < \phi \leq \frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$.

Here, $D^* = \frac{1}{6}\left(1+\sqrt{1+12r(1-\phi)}\right)$ and $\bar{\omega} = \frac{1}{3\phi}\left(-4+3\phi+\sqrt{1+12r(1-\phi)}\right)$, $0 < \bar{\omega} < 1$. Substitute in the bank objective function to get

$$W_B = \frac{144r(1-\phi)-(42r(1-\phi)+23)\sqrt{12r(1-\phi)+1}+31}{162(1-\phi)\phi}.$$

It is more convenient to consider $r < \frac{1}{4}$ and $r > \frac{1}{4}$ separately.

When $\bar{r}_{O_1} < r < \frac{1}{4}$, $\frac{d^2W}{d\phi^2} > 0$. Since the bank objective is convex, maximum is attained at one of the two boundaries. Direct comparison reveals that $W\left(\frac{2}{3}\left(2(1-2r)-\sqrt{(4r)^2-4r+1}\right)\right) > W\left(\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)\right)$. On the other hand, when $\frac{1}{4} < r < \frac{1}{2}$, $\frac{dW_i}{d\phi} > 0$. Since the bank objective is increasing in ϕ, maximum bank profit is attained at maximum relevant ϕ, i.e. $\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$.

Thus if the equilibrium level of bank opacity is in this region, we have

$$\phi^* = \begin{cases} \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right) & \text{if } \bar{r}_{O_1} < r < \frac{1}{4} \\ \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right) & \text{if } \frac{1}{4} < r < \frac{1}{2} \end{cases}$$

Case 4 : $\max\{\frac{3}{10}\left(1 - 2r + \sqrt{1-4r-6r^2}\right), \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)\} < \phi \leq 1.$

Here, D^* is the largest root of equation

$$-4D^3 + D^2(2r + \phi + 1) + r^2(\phi - 1) = 0,$$

and $\bar{\omega} = \frac{1}{\phi}\left(D^* - (1-\phi)(1 - \frac{r}{D^*})\right)$, $0 < \bar{\omega} < 1$. Bank profit is given by

$$W_B^{\text{case 4}} = W = \frac{3D^{*5} - 3D^{*4}(\phi+1) + D^{*3}\left(\phi^2 + \phi + 1\right) - r^3(1-\phi)^2}{6D^{*3}\phi}.$$

We will use $W_B^{\text{case 4}}$ for the objective function in this region since we use it to define the equilibrium thresholds.

We consider two cases separately, when $r < \bar{r}_{O_1}$ and when $r > \bar{r}_{O_1}$.

- $\mathbf{r < \bar{r}_{O_1}}$:

  In this region, $\frac{d^2 W^{\text{case 4}}}{d\phi^2} > 0,\ \forall\ \phi > \frac{3}{10}\left(1 - 2r + \sqrt{1-4r-6r^2}\right)$, thus the objective function is convex and the maximum is attained at one of the two boundaries. The upper boundary is $\phi = 1$ and the lower boundary is $\phi = \frac{3}{10}\left(1 - 2r + \sqrt{1-4r-6r^2}\right)$. The bank profit at the two boundaries is given by

$$W_B\left(\frac{3}{10}\left(1 - 2r + \sqrt{1-4r-6r^2}\right)\right) = \frac{1}{2} - \frac{8}{45}\sqrt{2\left(\sqrt{1-2r(3r+2)}+1\right) + r\left(3r + 6\sqrt{1-2r(3r+2)}+2\right)},$$

$$W_B(1) = \frac{(1-r)^2}{8}.$$

Where the first expression uses continuity of bank objective function on the boundary $\frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right)$, and case 2 above. Direct comparison of the two expressions reveals that the former expression is always larger than the latter. Thus in this range

$$\phi^* = \frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right)$$

- $\mathbf{r > \bar{r}_{O_1}}$:

  Consider the first order condition

$$\frac{dW_B^{\text{case 4}}(\phi, r)}{d\phi} = 0$$

The first order condition has either one or two solutions for $r \in (\frac{2}{15}, \frac{1}{2})$. Let ϕ_1 denote the larger solution. ϕ_1 exists for all $r \in (\frac{2}{15}, \frac{1}{2})$, $\phi_1 > \frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$, and $\frac{d^2W_B^{\text{case 4}}(\phi,r)}{d\phi^2} > 0$, i.e. ϕ_1 is a minimum.

Let ϕ_2 denote the smaller solution (if it exists). ϕ_2 exists only if $r > \hat{r} \in (\frac{2}{15}, \frac{1}{5})$, and $\frac{d^2W_B^{\text{case 4}}(\phi,r)}{d\phi^2} < 0$, i.e. ϕ_1 is a maximum. However, ϕ_2 is not always larger than $\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$, thus it is not always a relevant solution. Moreover, $\frac{d\phi_2}{dr} > 0$.

Let $\phi^*_{FOC} = \phi_2$, and let $\bar{r}_O^2$ denote the level of $r \in (\frac{1}{4}, \frac{1}{2})$ such that $\phi^*_{FOC}(r) = \frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$. Thus for $r > \bar{r}_{O_2}$, ϕ^*_{FOC} is an interior (local) maximum.

Given the above argument, for $r \in (\bar{r}_{O_1}, \bar{r}_{O_2})$, bank objective function is either decreasing or convex (or both) for $\phi \in \left(\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right), 1\right)$. Thus the maximum is attained at one of the two boundaries. The bank profit at the two boundaries is given by

$$W_B\left(\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)\right)=$$

$$\frac{46\sqrt{2r\left(r-\sqrt{r(r+10)+1}+5\right)+1}+2r\left(r-\sqrt{r(r+10)+1}+5\right)\left(7\sqrt{2r\left(r-\sqrt{r(r+10)+1}+5\right)+1}-24\right)-62}{9\left(-r+\sqrt{r(r+10)+1}-5\right)\left(-r+\sqrt{r(r+10)+1}+1\right)},$$

and

$$W_B(1)=\frac{(1-r)^2}{8},$$

where the first expression uses continuity of bank objective function on the boundary $\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$, and case 3 above. Direct comparison of the two expressions reveal that the former expression is larger than the latter when $r\in(\bar{r}_{O_1},\bar{r}_{O_2})$. Thus in case 4, in this range

$$\phi^*=\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$$

Next, let $\bar{r}_O$ denote $r\in(\bar{r}_{O_2},1)$ such that $W_B^{\text{case 4}}(\phi^*_{FOC},r)=\frac{(1-r)^2}{8}=W_B(1)$. For any $r\in(\bar{r}_{O_2},\bar{r}_O)$, bank surplus is first concave and then convex over the interval $\phi\in\left(\frac{1}{6}\left(1-r+\sqrt{r^2+10r+1}\right),1\right)$, with an interior (local) maximum and a larger interior (local) minimum. Thus the global maximum is obtained at either the local maximum, ϕ^*_{FOC}, or at the upper boundary $\phi=1$. Direct comparison of the corresponding levels of objective functions reveals that $W_B^{\text{case 4}}(\phi^*_{FOC}(r),r)>\frac{(1-r)^2}{8}=W_B(1)$ for $r\in(\bar{r}_{O_2},\bar{r}_O)$.

Finally, for $r>\bar{r}_O$, the objective function is larger at the corner $\phi=1$ compared to the interior local maximum, thus $\phi^*=1$.

Putting the cases together,

$$\phi^* = \begin{cases} \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right) & \text{if } 0 < r < \bar{r}_{O_1} \\ \frac{1}{6}\left(1 - r + \sqrt{r^2 + 10r + 1}\right) & \text{if } \bar{r}_{O_1} < r < \bar{r}_{O_2} \\ \phi^*_{FOC} & \text{if } \bar{r}_{O_2} < r < \bar{r}_O \\ 1 & \text{if } \bar{r}_O < r < \frac{1}{2} \end{cases}$$

Case 5 : $0 \leq \phi < 1 - \frac{1}{4r}$.

Here $\bar{\omega} = 1$, thus $W_B = 0$.

Comparison across cases. For each r, we compare the optimum across cases. Again it is easiest to treat 3 ranges separately.

1. $\mathbf{0 < r < \bar{r}_{O_1}}$:

   Here we compare the maximum across cases 1, 2, and 4. Case 1 shows that when $0 < r < \frac{2}{15}$, maximum is attained at $\phi = \frac{2}{5}$. Case 2 shows that $\phi = \frac{2}{5}$ is also optimal in that range. Thus within cases 1 and 2, $\phi = \frac{2}{5}$ is optimal.

   Case 4 argues that if $r < \bar{r}_{O_1}$, maximum bank profit is attained at $\phi = \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$. However, case 2 shows that $W_B(\frac{2}{5}) \geq W_B\left(\frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)\right)$. Thus the maximum is attained at $\phi = \frac{2}{5}$.

2. $\mathbf{\bar{r}_{O_1} < r < \frac{1}{4}}$:

   Here we compare the maximum across cases 1, 3, and 4. Case 1 shows that when $\frac{2}{15} < r < \frac{1}{4}$, maximum is attained at $\phi = \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)$, which also maximizes bank profit over the region covered by case 3. The latter implies $W_B\left(\frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)\right) \geq W_B(\frac{1}{6}\left(1 - r + \sqrt{r(r+10) + 1}\right)$.
   Since $\phi = \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$ maximizes bank profit over the region covered by Case 4, the maximum is attained at $\phi = \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)$.

3. $\mathbf{\frac{1}{4} < r < \frac{1}{2}}$:

Here we compare the maximum across cases 3 and 4. Case 3 shows that $W_B\left(\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)\right) \geq W_B\left(1-\frac{1}{4r}\right) = 0$. Comparing with case 4 in this region, and using continuity of the bank profit function at the boundary $\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$ yields

$$\phi^* = \begin{cases} \frac{1}{6}\left(1-r+\sqrt{r^2+10r+1}\right) & \text{if } \frac{1}{4} < r < \bar{r}_{O_2} \\ \phi^*_{FOC} & \text{if } \bar{r}_{O_2} < r < \bar{r}_O \\ 1 & \text{if } \bar{r}_O < r < \frac{1}{2} \end{cases}$$

Putting all the regions together leads the final result.

$$\phi^* = \begin{cases} \frac{2}{5} & \text{if } 0 < r < \bar{r}_{O_1} \\ \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2-4r+1}\right) & \text{if } \bar{r}_{O_1} < r < \frac{1}{4} \\ \frac{1}{6}\left(1-r+\sqrt{r^2+10r+1}\right) & \text{if } \frac{1}{4} < r < \bar{r}_{O_2} \\ \phi^*_{FOC} & \text{if } \bar{r}_{O_2} < r < \bar{r}_O \\ 1 & \text{if } \bar{r}_O < r < \frac{1}{2} \end{cases} \tag{A.10}$$

where ϕ^*_{FOC} is the solution to $\frac{dW_B^{\text{case 4}}(\phi,r)}{d\phi} = 0$ with $\frac{d^2W_B^{\text{case 4}}(\phi,r)}{d\phi^2}|_{\phi^*_{FOC}} < 0$. $\bar{r}_{O_2}$ is the value of $r \in \left(\frac{1}{4},\frac{1}{2}\right)$ such that $\phi^*_{FOC}(r) = \frac{1}{6}\left(1-r+\sqrt{r^2+10r+1}\right)$, $\bar{r}_{O_2} \approx 0.324$ and $\bar{r}_O > \bar{r}_{O_2}$ is the value of $r \in \left(\frac{1}{4},\frac{1}{2}\right)$ such that $W_B^{\text{case 4}}\left(\phi^*_{FOC}, r\right) = \frac{(1-r)^2}{8} = W_B(1)$, $\bar{r}_O \approx 0.477$.

□

Proof of Proposition 3

The proof involves two steps. First, we hold fixed the portfolio composition and solve for the bank's optimal portfolio allocation when she holds a transparent portfolio, an unobservable portfolio, and an opaque portfolio. The solution for this first step is given in the previous intermediate results. Specifically, Lemma A.1 and Lemma A.2 for the transparent case, Lemma A.3 and Lemma A.4 for the unobservable case, and Lemma A.7 and Lemma A.8 for the opaque case.

The second step is to solve for the bank's optimal choice of portfolio composition, given that she understands the optimal portfolio allocation associated with it. To do so, we compare the bank surplus,

$$W_B = \int_D^1 \left[(z - D) \frac{\partial H(z, \phi)}{\partial z} \right] dz,$$

when she holds the equilibrium transparent portfolio, the equilibrium unobservable portfolio, and the equilibrium opaque portfolio (step 1 in the proof).

We start by characterizing the bank surplus in each portfolio composition equilibrium.

Transparent Portfolio: Using the results in Lemma A.1 and Lemma A.2, the equilibrium is characterized as follows:

- For $r < \frac{1}{4}$, $\phi^*(r) = \frac{2+2r}{5}$ and $D^*(r) = \phi^*(r)$
- For $r \in [\frac{1}{4}, \bar{r}_T)$, $\phi^*(r) = \frac{2-2r}{3}$ and $D^*(r) = 1 - \phi^*(r)$
- For $r > \bar{r}_T$, $\phi^*(r) = 0$ and $D^*(r) > \phi^*(r)$.

Thus bank surplus is given by

$$W_B = \int_{D^*}^{1-\phi} \left[(z - D^*) \frac{\partial}{\partial z} \left(\frac{1}{1-\phi} (z - \frac{\phi}{2}) \right) \right] dz + \int_{1-\phi}^{1} \left[(z - D^*) \frac{\partial}{\partial z} \left(1 - \frac{(1-z)^2}{2\phi(1-\phi)} \right) \right] dz$$

Substituting for D^*,

$$W_B = \frac{1}{96\,(1-\phi)} \left(12\,(r-1)^2 + 12\phi\,(r-1) + 7\phi^2 \right)$$

We need to consider three cases: $r < \frac{1}{4}$, $\frac{1}{4} \le r < \bar{r}_T$, and $r \ge \bar{r}_T$

$\mathbf{r < \frac{1}{4}}$: The portfolio allocation is $\phi^*(r) = \frac{2+2r}{5}$. Substituting it into W_B,

$$W_B = \frac{28r^2 - 34r + 13}{90 - 60r}$$

$\mathbf{\frac{1}{4} \leq r < \bar{r}_T}$: The portfolio allocation is $\phi^*(r) = \frac{2-2r}{3}$. Substituting it into W_B,

$$W_B = \frac{2(1-r)^2}{18r + 9}$$

$\mathbf{r > \bar{r}_T}$: The portfolio allocation is $\phi^* = 0$. Substituting it into W_B,

$$W_B = \frac{12(r-1)^2}{96}$$

All Cases : All cases together yield the bank surplus

$$W_B^T = \begin{cases} \frac{28r^2-34r+13}{90-60r} & r < \frac{1}{4} \\ \frac{2(1-r)^2}{18r+9} & \frac{1}{4} \leq r < \bar{r}_T \\ \frac{1}{8}(r-1)^2 & r \geq \bar{r}_T \end{cases}$$

Unobservable Portfolio Using the results in Lemma A.3 and Lemma A.4, the equilibrium is characterized as follows:

- For $r < \bar{r}_{U_{low}}$, $\phi^*(r) = \frac{2}{5}$ and $D^*(r) = \phi^*(r)$
- For $r \in [\bar{r}_{U_{low}}, \bar{r}_U)$, $\phi^*(r) = \frac{3r}{2}$ and $D^*(r) = \frac{1}{2} - \frac{\sqrt{2-27r^2}}{2\sqrt{2}}$

We need to consider two cases: when $r < \bar{r}_{U_{low}}$ and when $\bar{r}_{U_{low}} \leq r < \bar{r}_U$.

$\mathbf{r < \bar{r}_{U_{low}}}$: Bank surplus is given by

$$W_B = \int_{D^*}^{1-\phi} \left[(z - D^*) \frac{\partial}{\partial z} \left(\frac{1}{1-\phi} (z - \frac{\phi}{2}) \right) \right] dz + \int_{1-\phi}^{1} \left[(z - D^*) \frac{\partial}{\partial z} \left(1 - \frac{(1-z)^2}{2\phi(1-\phi)} \right) \right] dz$$

Substituting for D^*,

$$W_B = \frac{12 - 12\phi + 7\phi^2}{96(1-\phi)}$$

Substituting for ϕ^*,

$$W_B = \frac{13}{90}$$

$\mathbf{\bar{r}_{U_{low}} \leq r < \bar{r}_U}$**:** Bank surplus is given by

$$W_B = \int_{D^*}^{\phi} \left[(z - D^*) \frac{\partial}{\partial z} \left(\frac{z^2}{2\phi(1-\phi)} \right) \right] dz + \int_{\phi}^{1-\phi} \left[(z - D^*) \frac{\partial}{\partial z} \left(\frac{1}{1-\phi}(z - \frac{\phi}{2}) \right) \right] dz$$
$$+ \int_{1-\phi}^{1} \left[(z - D^*) \frac{\partial}{\partial z} \left(1 - \frac{(1-z)^2}{2\phi(1-\phi)} \right) \right] dz$$

Substituting for D^*,

$$W_B = \frac{1}{2} - \frac{8}{9}\sqrt{\frac{2}{3}}\sqrt{(1-\phi)\phi}$$

and for ϕ^*,

$$W_B = \frac{1}{6}(3 - 8r)$$

All Cases : All cases together yield the bank surplus

$$W_B = \begin{cases} \frac{13}{90} & r < \bar{r}_{U_{low}} \\ \frac{1}{2} - \frac{16\sqrt{(r+1)\left(1 - \frac{2(r+1)}{5}\right)}}{9\sqrt{15}} & \bar{r}_{U_{low}} \leq r < \bar{r}_U \\ \frac{1}{6}(3 - 8r) & r \geq \bar{r}_U \end{cases}$$

Opaque Portfolio Using the results in Lemma A.7 and Lemma A.8, the equilibrium characterization is as follows. We need to consider five cases.

$\mathbf{r < \frac{2}{15}}$**:** Portfolio allocation is $\phi^* = \frac{2}{5}$ and bank surplus

$$W_B = \frac{13}{90}$$

$\mathbf{\frac{2}{15} < r < \frac{1}{4}}$**:** Portfolio allocation is $\phi^* = \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)$ and bank surplus

$$W_B = \frac{23\sqrt{4r\left(8r+2\sqrt{(2r-1)^2}-1\right)+1}+2r\left(8r+2\sqrt{(2r-1)^2}-1\right)\left(7\sqrt{4r\left(8r+2\sqrt{(2r-1)^2}-1\right)+1}-24\right)-31}{36\left(4r+\sqrt{(2r-1)^2}-2\right)\left(8r+2\sqrt{(2r-1)^2}-1\right)}$$

$\mathbf{\frac{1}{4} < r < r_z \approx 0.324}$**:** Portfolio allocation is $\phi^* = \frac{1}{6}\left(1 - r + \sqrt{r^2 + 10r + 1}\right)$ and bank surplus

$$W_B = \frac{46\sqrt{2r\left(r - \sqrt{r(r+10)} + 5 + 1\right)}\left(7\sqrt{2r\left(r - \sqrt{r(r+10)} + 5 + 1\right)} + 1 - 24\right) - 62}{9\left(-r + \sqrt{r(r+10)} + 1 - 5\right)\left(-r + \sqrt{r(r+10)} + 1 + 1\right)}$$

$\mathbf{r_z < r < r_h \approx 0.477}$**:** Portfolio allocation is $\phi^* = \phi^*_{FOC}$ and bank surplus

$$W_B = \frac{(1-r)^2}{8}$$

$\mathbf{r > r_h}$**:** Portfolio allocation is $\phi^* = 1$ and bank surplus

$$W_B = \frac{(1-r)^2}{8}$$

All Cases: All cases together yields the bank surplus

$$W_B = \begin{cases} \frac{13}{90} & r < \bar{r}_{O_1} \\ \frac{23\sqrt{4r+\sqrt{(2r-1)^2}-1}+2r\left(8r+2\sqrt{(2r-1)^2}-1\right)\left(7\sqrt{4r+\sqrt{(2r-1)^2}-1}-23\right)-20}{36\left(4r+\sqrt{(2r-1)^2}-2\right)\left(8r+2\sqrt{(2r-1)^2}-1\right)} & \bar{r}_{O_1} \leq r < \frac{1}{4} \\ \frac{46\sqrt{2}\sqrt{r\left(r-\sqrt{r(r+10)}+6\right)}\left(7\sqrt{2}\sqrt{r\left(r-\sqrt{r(r+10)}+6\right)}-23\right)-62}{9\left(-r+\sqrt{r(r+10)}-4\right)\left(-r+\sqrt{r(r+10)}+2\right)} & \frac{1}{4} \leq r < \bar{r}_{O_2} \\ \frac{1}{8}(1-r)^2 & r \geq \bar{r}_{O_2} \end{cases}$$

With the surplus in hand, we proceed to compare them.

Bank Surplus Comparison We need to consider several cases depending on the depositors' outside value r.

First, let $r < \bar{r}_{O_1}$. It holds that

$$W_B^T \leq W_B^O = W_B^U$$

Thus the bank prefers the opaque or the unobservable portfolio against the transparent one.

Next let $\bar{r}_{O_1} \geq r < \frac{1}{4}$. It holds that

$$W_B^T < W_B^O \leq W_B^U$$

Thus the bank prefers the unobservable portfolio against the opaque and transparent ones.

Next let $\frac{1}{4} \leq r < \bar{r}_{U_{low}}$. It holds that

$$W_B^T < W_B^O \leq W_B^U$$

Thus the bank prefers the unobservable portfolio against the opaque and transparent ones.

Next let $\bar{r}_{U_{low}} \leq r < \bar{r}_U$. It holds that

$$W_B^T < W_B^O \leq W_B^U$$

Thus the bank prefers the unobservable portfolio against the opaque and transparent ones.

Next let $\bar{r}_U \leq r < \bar{r}_{O_2}$. It holds that

$$W_B^U < W_B^T < W_B^O$$

Thus the bank prefers the opaque portfolio against the unobservable and transparent ones.

Next let $\bar{r}_{O_2} \leq r < \bar{r}_T$. It holds that

$$W_B^U < W_B^T < W_B^O$$

Thus the bank prefers the opaque portfolio against the unobservable and transparent ones.

Next let $\bar{r}_T \leq r < \bar{r}_O$. It holds that

$$W_B^U < W_B^T < W_B^O$$

Thus the bank prefers the opaque portfolio against the unobservable and transparent ones.

Lastly let $r \geq \bar{r}_O$. It holds that

$$W_B^U < W_B^T = W_B^O$$

Thus the bank prefers the opaque or the transparent portfolio against the unobservable one.

We conclude the following. The bank chooses to hold the unobservable portfolio whenever it exists since it delivers the (weakly) higher surplus. This is the case when r is relatively low, i.e $r < \bar{r}_U$. From Lemma A.4, we know that $\phi^*(r) \in (0,1)$ and thus the bank diversifies. For intermediate levels of r, i.e. $\bar{r}_U \leq r < \bar{r}_O$, the bank holds the opaque portfolio since it delivers a strictly higher surplus than the transparent portfolio. From Lemma A.8, we know that $\phi^*(r) \in (0,1)$ and thus the bank diversifies. When r is too high, i.e. $r > \bar{r}_O$, the bank holds a transparent portfolio due to two reasons. First, the opaque and transparent portfolios deliver the same surplus. Second, from Lemma A.2 and Lemma A.8, we know that these portfolios are not diversified, $\phi^*(r) = 1$, and the bank only holds a single transparent project.

B Bank Portfolio Allocation Trade-off with an Opaque Portfolio

To prove Example 1 in Section 5, we start by proving intermediate results about how the optimal deposit rate and optimal withdrawal probability behave as bank's portfolio allocation choice ϕ changes.

Behavior of D^* and ω^* as ϕ changes In the following lemmas, we use the results in Lemma A.7 and Lemma A.8.

Lemma B.9 (Behavior of the optimal deposit rate D^*)**.** *For $r < \bar{r}_{O_1}$, bank's optimal portfolio allocation ϕ^* minimizes D^*. For $\bar{r}_{O_1} < r < \frac{1}{4}$, bank's optimal portfolio allocation ϕ^* does not minimize D^*. For $\frac{1}{4} < r < \bar{r}_{O_2}$, bank's optimal portfolio allocation ϕ^* minimizes D^*. For $r \geq \bar{r}_{O_2}$, bank's optimal portfolio allocation ϕ^* does not minimize D^*.*

Proof. Consider the optimal deposit rate D^* when the bank holds an opaque. We need to look into different cases. For $r < \bar{r}_{O_1}$, D^* can take two values. First, $D^* = \frac{2-\phi}{4}$, which is decreasing in ϕ. The optimal portfolio is $\phi^* = \frac{2}{5}$, the upper bound in this case. Thus bank's choice minimizes D. Second, $D^* = \sqrt{\frac{2}{3}\phi(1-\phi)}$ which is increasing in ϕ. The optimal portfolio is $\phi^* = \frac{2}{5}$, the lower bound in this case. Thus bank's choice minimizes D.

For $\bar{r}_{O_1} < r < \frac{1}{4}$, $D^* = \frac{1}{6}\left(\sqrt{12r(1-\phi)+1}+1\right)$ which is decreasing in ϕ. The optimal portfolio is $\phi^* = \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)$, the lower bound in this case. Thus bank's choice does not minimize D.

For $\frac{1}{4} < r < \bar{r}_{O_2}$, also $D^* = \frac{1}{6}\left(\sqrt{12r(1-\phi)+1}+1\right)$ which is decreasing in ϕ. The optimal portfolio is $\phi^* = \frac{1}{6}\left(1-r+\sqrt{r^2+10r+1}\right)$, the upper bound in this case. Thus bank's choice minimizes D.

For $\bar{r}_{O_2} < r < \bar{r}_O$, $D^* = D_3$ where D_3 is the largest root of

$$P(D;\phi,r) := -4D^3 + D^2(1+\phi+2r) - r^2(1-\phi) = 0,$$

in Lemma A.7 Case 4. Assume that (ϕ, r) satisfies the conditions of Case 4 in Lemma A.8:

$$r \in (\bar{r}_{O_2}, 0.5), \quad \phi \in \left(\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right), 1\right).$$

Then, the partial derivatives of $P(D;\phi,r)$ are:

$$\frac{\partial P}{\partial \phi} = D^2 + r^2 > 0,$$
$$\frac{\partial P}{\partial D} = -12D^2 + 2D(1+\phi+2r).$$

By the implicit function theorem, the derivative of D_3 with respect to ϕ is

$$\frac{dD_3}{d\phi} = -\frac{\partial P/\partial \phi}{\partial P/\partial D}.$$

From Lemma A.8, we know that D_3 is a strict local maximum in the range of (ϕ, r) considered. This ensures that $\frac{\partial P}{\partial D}(D^*, \phi, r) < 0$. Since the numerator is positive and the denominator is negative, we conclude that:

$$\frac{dD^*}{d\phi} > 0.$$

Hence, $D^*(\phi, r)$ is strictly increasing in ϕ when $\bar{r}_{O_2} < r < \bar{r}_O$. From Lemma A.8 we know that the optimal portfolio is $\phi^* = \phi^*_{FOC}$, a local maximum in this case. Thus, the bank's choice does not minimize D.

And $r > \bar{r}_O$, $D^* = D_3$ again which is increasing in ϕ. From Lemma A.8 we know that the optimal portfolio is $\phi^* = 1$, the upper bound in this case. Thus, the bank's choice does not minimize D.

All cases together deliver the lemma. □

Lemma B.10 (Behavior of the optimal withdraw probability ω^*)**.** *For $r < \bar{r}_{O_2}$, bank's optimal portfolio allocation ϕ^* minimizes ω^*. For $\bar{r}_{O_2} < r < \bar{r}_O$, bank's optimal portfolio allocation ϕ^* does not minimize ω^*. For $r > \bar{r}_O$, bank's optimal portfolio allocation ϕ^* minimizes ω^*.*

Proof. Consider the optimal withdrawal probability ω^* when the bank holds an opaque portfolio. We need to look into different cases.

For $r < \bar{r}_{O_1}$, $\omega^* = 0$. Thus any choice of the bank minimizes the withdrawal probability.

For $\bar{r}_{O_1} < r < \frac{1}{4}$, $\omega^* = \frac{1}{3\phi}\left(-4 + 3\phi + \sqrt{1 + 12r(1-\phi)}\right)$, which is increasing in ϕ. The optimal portfolio is $\phi^* = \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)$, the lower bound in this case. Thus bank's choice minimizes ω.

For $\frac{1}{4} < r < \bar{r}_{O_2}$, also $\omega^* = \frac{1}{3\phi}\left(-4 + 3\phi + \sqrt{1 + 12r(1-\phi)}\right)$ which is increasing in ϕ. The optimal portfolio is $\phi^* = \frac{1}{6}\left(1 - r + \sqrt{r^2 + 10r + 1}\right)$, the upper bound in this case. Thus bank's choice does not minimize ω.

For $\bar{r}_{O_2} < r < \bar{r}_O$, $\omega^* = \frac{1}{\phi}\left(D^* - (1-\phi)(1 - \frac{r}{D^*})\right)$ where $D^* = D_3$ (Case 4 of Lemma A.7). ω^* is increasing in $D^*(\phi)$ and is decreasing in ϕ for $D > r$. From Lemma B.9, we know that $D^*(\phi)$ is increasing in ϕ for $r \in (\bar{r}_{O_2}, 0.5)$.

We analyze the derivative of $\bar{\omega}$ with respect to ϕ, using the chain rule and the fact that $D^* = D_3$ depends on ϕ. We rewrite:

$$\bar{\omega}(\phi, r) = \frac{1}{\phi}\left(D^* - 1 + \phi + \frac{r(1-\phi)}{D^*}\right) = \frac{D^* - 1}{\phi} + 1 + \frac{r(1-\phi)}{\phi D^*}.$$

Differentiating,

$$\frac{\partial \bar{\omega}}{\partial \phi} = -\frac{1}{\phi^2}\left(D^* - (1-\phi)\left(1 - \frac{r}{D^*}\right)\right) + \frac{1}{\phi}\left(\frac{dD^*}{d\phi} + \left(1 - \frac{r}{D^*}\right) - \frac{(1-\phi)r}{D^{*2}} \cdot \frac{dD^*}{d\phi}\right).$$

We analyze the sign of this expression for $r \in (\bar{r}_{O_2}, 0.5)$. The first term is strictly negative. Since $D^* > r$ and $\phi > 0$, the expression inside the parentheses is positive, so the whole term is negative. The second term is positive, but each of its components is bounded. Specifically, $\frac{dD^*}{d\phi} > 0$ by Lemma B.9, but the term $-\frac{(1-\phi)r}{D^{*2}} \cdot \frac{dD^*}{d\phi}$ reduces the overall magnitude of the derivative. Over the domain $\phi \in (0.63, 1)$ and $r \in (\bar{r}_{O_2}, 0.5)$, the negative first term dominates. As $\phi \to 1$, the term $\frac{r(1-\phi)}{\phi D^*}$ vanishes, and $\bar{\omega} \to D^*/\phi \to 1$, but with a strictly negative slope.

Hence, the total derivative is strictly negative $\frac{\partial \bar{\omega}}{\partial \phi} < 0$, and $\bar{\omega}(\phi, r)$ is strictly decreasing in ϕ for $r \in (\bar{r}_{O_2}, 0.5)$.

The optimal portfolio is $\phi^* = \phi^*_{FOC}$, a local maximum in this case, which is smaller than the upper bound 1. Thus, the bank's choice does not minimize ω.

Lastly, for $r > \bar{r}_O$, $\omega^* = \frac{1}{\phi}\left(D^* - (1-\phi)(1 - \frac{r}{D^*})\right)$ again decreasing in ϕ. The optimal portfolio is $\phi^* = 1$, the upper bound in this case. Thus bank's choice minimizes ω.

All cases together deliver the lemma. □

Proof of Example 1 Lemma B.9 and Lemma B.10 together deliver the result in Example 1, which is also depicted in Figure 4.